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# Numerical and Mathematical modeling of dynamic thermal behavior of building

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## Abstract
The comfortable habitat is necessitating the improvement of energy efficiency directly related to the heat. For this reason, this work is essentially focused on numerical and mathematical modeling of building dynamic thermal in the two-dimensional behavior. A zonal model is chosen for heat and mass transfer modeling. This approach, used to describe the temperature and air velocity parameters in the building with a very fine degree of accuracy However, the equations of continuity, momentum and energy are already known to be the main physical problem based on primitive variables without giving analytical solutions. Indeed, numerical methods proved their importance while the ADI "Alternating Direction Implicit" Method is used in this work, and our findings showed as expecting various variations of the air speed dependently to heat spread in different instants with an interesting dynamic thermal behavior. These results are confirmed by other studies encouraging the implementation of new subjects in this field.

## Nomenclature

$\delta_{i,j}$ : Kronecker Symbol
$\rho$ : Density
$\mu$ : Dynamic viscosity of the fluid
$\lambda$ : Thermal conductivity
$\Phi$ : function of heat dissipation
$C_p$ : Heat capacity
$P$ : Pressure
$T$ : Temperature
$v_x$ : Component of speed vector along the axis (ox)
$v_z$ : Component of speed vector along the axis (oz)

## 1. Introduction

At all times, power transmission problems to buildings, especially in the heat, had a decisive importance for the maximum efficiency study for minimum energy expenditure. Indeed, the buildings' envelope is designed to act as a thermal barrier to recreate an independent interior microclimate of weather fluctuations [12]. All the "heterogeneous" aspects in this study have given special attention because it has shed light on a little discussed concept in the study of buildings and cities at different levels such as thermal comfort at the city level, district, in construction and in material levels [6].
For example, a building under wind action will be represented too completely different, depending on the objectives. The change in speed and direction of the wind on the edge of urban building makes it difficult [11].
Among the six traditional parameters of the building thermal comfort is the air speed, which influences the heat transfer by convection in the building. Overall, approximately 60% of heat losses are convection with ambient air, 35% by radiation and 1% by conduction with the ground. We note that the exchanges by radiation are important explaining why we are also sensitive to the walls' temperature.
We define, for example, thermal comfort as a state of mind that expresses satisfaction to the thermal environment [5]. This parameter can be quantified through experimentation on occupants or models, which establish empirical physical models.
Models describing the thermal behavior of buildings provide insight and design envelope in order to achieve lower energy consumption while complying with minimum standards for thermal comfort. However, the study of real buildings is very difficult, we build their mathematical models. Similarly, it is impossible to draw up a comprehensive model representing a building in its entirety. Each model therefore represents only one or some aspects of reality, according to the objectives of the study [8]. The originality of the present work is to establish a numerical and mathematical modeling to study the flow of short fibers in a rigid cylinder by taking in account

the non Newtonian character of the fluid and a non permanent flow. Te obtained equations are solved using a finite difference method [4].

## 2. Mathematical Formulations

The movement of the compressible fluid in question in a Cartesian coordinate system is described by the following equations: equation of continuity, momentum equation and the energy equation [2], [3], [4].

2.1 *Continuity Equation*

The continuity equation is written as:

$$\frac{d\rho}{dt} = -\rho \mathrm{div}(v) \tag{1}$$

On the other hand, in the case of a low flow Mach number [13], it can be considered that the variation in density of the fluid as a function of time is negligible, either:

$$\mathrm{div}(v) = 0 \tag{2}$$

*2.2 Equation of momentum*

The momentum equation can be written as:

$$\rho\frac{\partial(\vec{v})}{\partial t} + \rho(\vec{v}.\nabla).\vec{v} = -\overrightarrow{\mathrm{div}}(p) + \rho.\vec{f} + \overrightarrow{\mathrm{div}}(\tau) \tag{3}$$

For a Newtonian fluid, the viscosity forces can be expressed in terms of dynamic viscosity μ:

$$\tau_{i,j} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \delta_{i,j}\left(\frac{2}{3}\mu - K_m\right)\nabla.v \tag{4}$$

For the air, which is of low density, the term $K_m$ mass viscosity is negligible, [14].

2.3 *Equation of energy conservation*

The equation of conservation of energy as temperature is [13], [14] written as:

$$\rho C_p\frac{DT}{Dt} = \lambda\nabla^2 T + \mu F_\mu + \rho T\left(\frac{\partial\left(\frac{1}{\rho}\right)}{\partial T}\right)\frac{DP}{Dt} \tag{5}$$

Avec:

$$\mu F_\mu = \frac{1}{2}\mu\left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}(\nabla.v)\delta_{i,j}\right]^2 \tag{6}$$

## 3. Numerical Method

The equations obtained above are non-linear and strongly coupled, so they do not admit analytical solutions under very simplified. A numerical solution is needed to be able to determine the velocity profile and temperature [15-28].

*3.1 Resolution Process*

The resolution method is that of ADI (Alternating Direction Implicit). It consists of iterating the implicit system of equations in two stages. The first step is to compute the solution between time t = 0. Moreover, t = n + 1/2 by implementing an implicit method and an explicit method x z. The second step changed the time n + 1/2 n + 1 by implementing this time, an explicit and implicit method in x z [8]-[11].

The equation of momentum is written:

According to the axis (Ox):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}\right) \tag{7}$$

According to the axis (Oz):

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z}\right) \tag{8}$$

The fluid is considered Newtonian and quasi-compressible, the shear stress tensor is written in the following form:

$$\tau = \left(\varepsilon - \frac{2}{3}\mu\right)\mathrm{div}\vec{v}.\delta + 2\mu D \tag{9}$$

Or:

$$\tau = 2\mu\frac{1}{2}[\mathrm{div}\vec{v} + (\mathrm{div}\vec{v})^{T}] = 2\mu D \quad (10)$$

The equation of the projection of the momentum gives:
According to the axis (ox):

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + \frac{R}{M}\left(\frac{\partial T}{\partial x}\right) - \frac{2\mu}{\rho_0}\frac{\partial^2 v_x}{\partial x^2} = \frac{\mu}{\rho_0}\frac{\partial^2 v_x}{\partial x\,\partial z} + \frac{\mu}{\rho_0}\frac{\partial^2 v_z}{\partial z^2} - v_z\frac{\partial v_x}{\partial z} \quad (11)$$

According to the axis (oz):

$$\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial z} + \frac{R}{M}\left(\frac{\partial T}{\partial x}\right) - \frac{2\mu}{\rho_0}\frac{\partial^2 v_z}{\partial z^2} = \frac{\mu}{\rho_0}\frac{\partial^2 v_z}{\partial x\,\partial z} + \frac{\mu}{\rho_0}\frac{\partial^2 v_x}{\partial x^2} - v_x\frac{\partial v_z}{\partial x} \quad (12)$$

The equation of the projection of the momentum gives:
According to the axis (ox):

**The first step**

This method allows ADI to explain the discretized equations and propose solutions in the following form:

$$v_{x(i,j)}^{k+1/2} = \rho_{(i,j)} v_{x(i+1,j)}^{k+1/2} + \beta_{(i,j)} \quad (13)$$

$$v_{x(i-1,j)}^{k+1/2} = \rho_{(i-1,j)} v_{x(i,j)}^{k+1/2} + \beta_{(i-1,j)} \quad (14)$$

**The second step**

In the second half step, the discretization proposed by this first method is of the following form:

$$v_{x(i,j)}^{k+1} = \rho_{(i,j)} v_{x(i+1,j)}^{k+1} + \beta_{(i,j)} \quad (15)$$

$$v_{x(i,j-1)}^{k+1} = \rho_{(i,j-1)} v_{x(i,j)}^{k+1} + \beta_{(i,j-1)} \quad (16)$$

The equation of the projection of the momentum gives:
According to the axis (oz):

**The first step**

Based on the same ADI method, the discretized equations and propose solutions are noticed in the following form:

$$v_{z(i,j)}^{k+1/2} = \rho_{(i,j)} v_{z(i,j+1)}^{k+1/2} + \beta_{(i,j)} \quad (17)$$

$$v_{z(i,j-1)}^{k+1/2} = \rho_{(i,j-1)} v_{z(i,j)}^{k+1/2} + \beta_{(i,j-1)} \quad (18)$$

**The second step**

Indeed, from the discretization proposed by this first method, we could conclude:

$$v_{z(i,j)}^{k+1} = \rho_{(i,j)} v_{x(i,j)}^{k+1} + \beta_{(i,j)} \quad (19)$$

$$v_{z(i-1,j)}^{k+1} = \rho_{(i-1,j)} v_{z(i,j)}^{k+1} + \beta_{(i-1,j)} \quad (20)$$

The equation of energy conservation:

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_z\frac{\partial T}{\partial z}\right) = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (21)$$

**The first step**

The ADI method led us to explain the discretized equations and propose solutions as following:

$$T_{(i,j)}^{k+1/2} = \rho_{(i,j)} T_{(i+1,j)}^{k+1/2} + \beta_{(i,j)} \quad (22)$$

$$T_{(i-1,j)}^{k+1/2} = \rho_{(i-1,j)} T_{(i+1,j)}^{k+1/2} + \beta_{(i-1,j)} \quad (23)$$

**The second step**

Following the first part described above, the second half step is written as:

$$T_{(i,j)}^{k+1} = \rho_{(i,j)} T_{(i+1,j)}^{k+1} + \beta_{(i,j)} \quad (24)$$

$$T_{(i,j-1)}^{k+1} = \rho_{(i,j-1)} T_{(i,j)}^{k+1} + \beta_{(i,j-1)} \quad (25)$$

Recurrence formulas $\alpha_{(i,j)}$ and $\beta_{(i,j)}$ are primed using the data $\alpha_{(1)}$ and $\beta_{(1)}$ from the boundary conditions. During the first scan $i = 2$ from up $i = i_{\max}$ , the functions $\alpha$ and $\beta$ are determined. The physical quantity will be calculated during the reverse scan $i = i_{\max-1}$ from up $i = 2$ .

*3.2 Boundary conditions*

We consider the boundary conditions:

3.2.1 *Dynamic Conditions:*

For: $0\langle x\langle l_x$ and, z=0 $v_x = 0.4$

And for: $0\langle x\langle l_x$ and, $z = l_z$ $v_x = 0$

Also for: $0\langle z\langle l_z$ and, $x = 0$ $v_z = 0$

And for: $0\langle z\langle l_z$ and, $x = l_x$ $v_z = 0.1.z^2$

*3.2.2 Thermal conditions*

For: x=0, T=T2

And for: x=L, T=T3

Also for: z=0, T=T1

And for: z=H, T=T0

3.2.3 *Convergence test*

We consider the boundary conditions:

The test of convergence of the solution of the problem studied concerns the temperature that requires checking [27-34]:

$$Sup\left|\frac{T_m(i,j,t) - T_{m+1}(i,j,t)}{T_{m+1}(i,j,t)}\right| < \varepsilon$$

## 4. Results and Discussion

Figure 1 shows the profile of the speed in the axial direction variation for different times. One can remark that the values of this speed are very low. We can explain these results by the fact that the flow is gouverned by the gravity; it is why the speed is very low horizontaly. We can also observe a perturbation of the profile during time. This can be explained by the outside excitation. In figure 2, it is reported the profile of speed in the vertical direction. One can see a quik increase between the start of the phenomenen and 0.8 s followed by a decrease until zero.

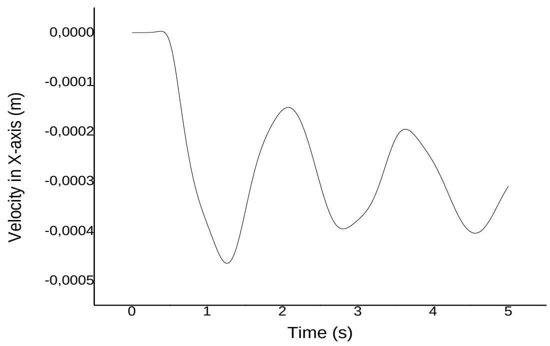


**Figure 1:** Evolution of the velocity along the axis (ox)

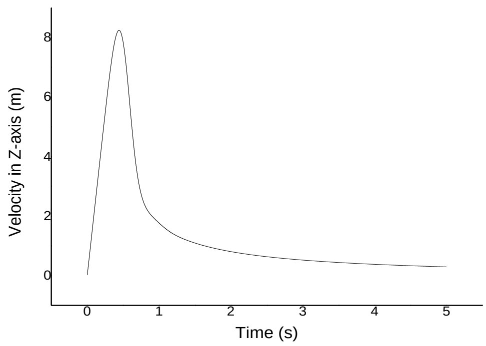


**Figure 2**: Evolution of the velocity along the axis (oz)

Figure 3 illustrates the profile of the temperature as function of time. We can remark a slow increase from two seconds. We can explain this result by the fact that the room is isulating and the air needs time to be influenced by the heat, and one time it is heated, the temperature increases slowlly because ther is no lost of heat from the walls.

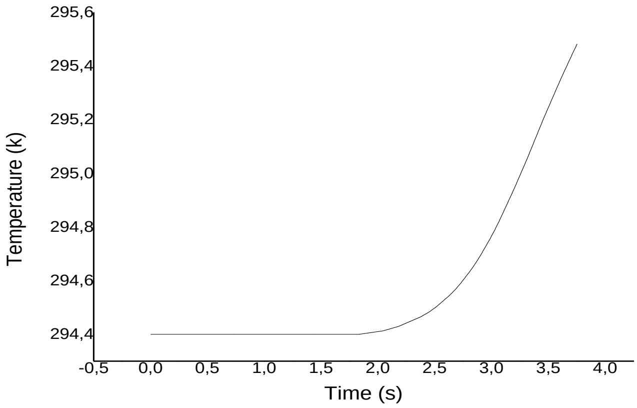


**Figure 3**: Profile of the temperature

In figure 4 is reported the profiles of the speed as function of depth for four specific times (the quarter, the half, the three quarters and the full period). For each time, we remark that the speed increases with depth. This increase is due to the effect of gravity. A comparison of the profiles confirms the fact that the speed decreases when time increases. One can also remark that the maximum speed of the fluid (air) is less than 0.25 m/s. We can conclude that the thermal controls are satisfied.

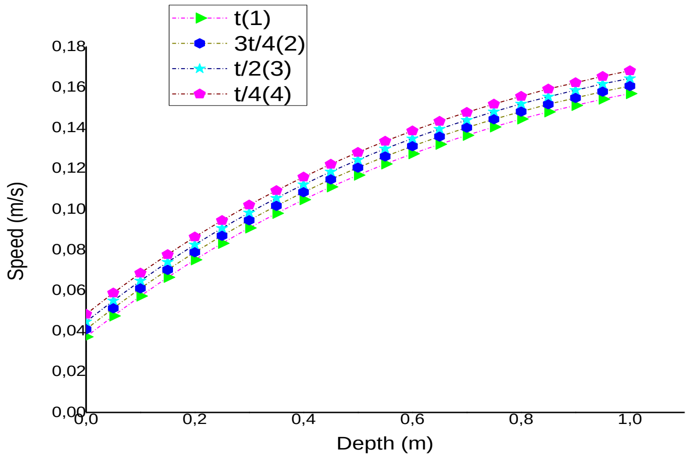


**Figure 4**: Evolution of the speed in two dimensions

Figure 5 shows the profile of the temperature as function of depth. We can easily remark that the temperature decreases rapidly at the highest positions due to the gravity combiend of the effect of the source term. Starting from z = 0.05 m, the profile of the temperature became uniform. We can explain the result by the fact: for low positions the profile of temperature is constant.

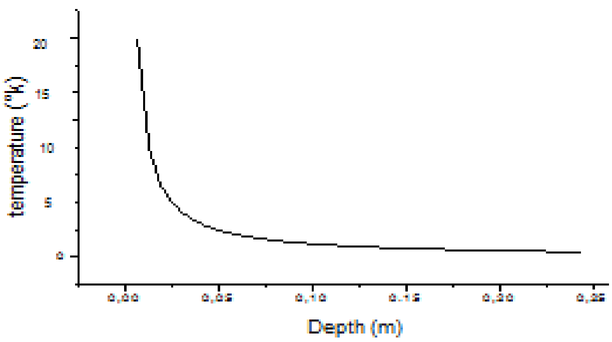


**Figure 5**: Temperature along the axe (oz)

## Conclusions

In this work, we presented different results of principal key parameters for evaluating the energy efficiency in housing. The suggested model by the boundary conditions gives an air speed control and thermal behavior in the building in two-dimensional case for proper human comfort as needed. Indeed, the comfort is directly related to the air diffusion quality into the room, to ensure a real speed under 0.25 m / s.

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