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New approach applied to analyzing a periodic Helmholtz resonator

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Abstract

Because of the population growth, high-rise building and heavy traffic, modern big cities and towns suffer from the increasing noise pollution problem. Therefore, reduction of noise becomes an important and challenging task for the sustainable development of modern cities. Meanwhile, industrial product such as machinery, home appliances and air moving devices need to be used significantly quieter because of customer's requirement and competition among manufactures. As a result, noise problem becomes more and more important and a new noise abasement technology should be developed to achieve the required environment or to improve the product quality. Noise control is, however, not a simple task. To adopt effective and economic control solutions, engineers must be able to understand the principles of sound generation and transmission as well as available noise control strategies. There are several ways to reduce noise, and the Helmholtz resonators (HR) are one of the most used filters.The acoustic wave propagation in periodic structures has attracted growing interest in recent decades. Many periodic structures were studied and various theoretical approaches were used. All have highlighted the existence of physical properties such as the presence of forbidden bands corresponding to high attenuation, and bandwidth corresponding to a negligible attenuation. In this work, we study a periodic structure formed by Helmholtz resonators (HR). We use two methods of analysis: the Transfer Matrix Method (TMM) and the Function Interface Response (FIR) or Green's Function Method (GFM). We note at the outset that these methods lead to the same results.

1. Introduction

The noise produced by several sources (automobile engine, aeroplane reactor, industrial pipeline,...) is a real problem which affects the quality of our life and constitutes true polluting environment. Helmholtz resonators (called simply resonators or HR, hereafter) are devices with a resonance peak designed to control noise. They are useful against noise centralized in a narrow frequency band. Many studies have tried to accurately predict resonant frequency. The Helmholtz resonator (HR) is a filter which is often used for the reduction of sound. A periodic structure is composed of a number of identical structural components that are joined together to form a whole complex. Periodic structures can be classified in general into three categories: (1) the periodic medium, (2) the periodically inhomogeneous medium and (3) the periodically bounded medium. There are several methods which can be used to study the problem. Our contribution is firstly to reduce the noise by using Matrix Transfer Method (MTM) and secondly to propose a new approach based on Response Interface Function (RIF) or Green function method [1]. We apply these methods to analyzing a periodic acoustic Helmholtz resonator. Knowing that the HR is a selective filter, the pass-band is very small. Because a single resonator has a narrow resonance peak, combining several resonators is a possible way to obtain a broader band of noise attenuation. However, to investigate the unusual attenuation of sound transmission in the periodic structure at low frequencies, the distance between two nearby resonators should be larger. In the present study, this distance between two nearby resonators is much larger than the dimension of the resonators. It is hoped that the present study can provide a stepping stone for the investigation of the acoustic properties of ducts loaded periodically

with resonators and its potential application in noise control. We consider a periodic structure formed by identical HR and we show that there are the stop band and the pass band. The problem is taken for one dimensional geometry and without losses. Each cell of the structure is formed by the waveguide with rigid wall. Under these conditions when we use the MTM, only the plane mode is taken into account. Several authors applied this method to highlight the formation of the forbidden band [2, 3, 4]. Wang [5] uses this method and gives the dispersion relation but without demonstration. In this context, our approach is to give some details on calculations of the relation dispersion and the transmission loss (TL). We find that the methods using RIF andMMT in this work lead to a good concordance between both methods.
The effect of periodicity in a structure was observed firstly by Brillouin [6] who identified the bands of frequencies and bandwidths of stops. The studies of the characteristics of one-dimensional periodic structures have been widely reported because they are so easy to analyses because of the simplicity of the geometry. Mead [7] has considered the periodic structure as a structure constituted basically of several identical structural components that are joined together to form a continuous structure. Gupta [3] studied a mechanical periodic structure and considered that the structure is equivalent at an association of identical cell. The basic idea of periodic structure [1-5] is that when a wave propagates in a network and encounters a discontinuity in the geometry of the cell, a part of the wave will propagate through the cell region and another portion will be reflected back to the previous cell. In a regular structure, the wave is assumed to propagate without energy losses to the limits of this structure. It is proposed in this paper to study various periodicals networks all dealing with dimensional geometry. We mainly use one dimensional geometry and we use plane wave assumption to analyze the acoustic performances of periodicHR. In this case, only the acoustics plane mode, common to different geometries, can be propagated and the shape of the geometry it's not important. At low frequencies of interest in the present study, the geometrical dimensions considered here are significantly smaller than therelatively long wavelengths.
Finally, we note that the motivating interest in this study is that the waves which occur in periodic media, known as Bloch waves (Kittel, [9]**)**, have very unusual dispersion and attenuation characteristics. The present analytic study concentrates on the acoustic behavior characteristics of one-dimensional pipe periodicallyinstalling simple Helmholtz resonators (fig.1).

2. Analytical approaches

The object of this study is to present theoretically a detailed study of the transmission loss coefficient and the dispersion relation. We present and discuss the band structure with various physical parameters. The model and method of calculation are presented, also the numerical illustrations as well as the discussion of the dispersion curves is given. The following figure shows a main guide which is connected periodically Helmholtz resonators. Obviously, it also shows a view of the periodic structure with simple 1D HR.The period of the structure is denoted by "d_1"witch represents the distance between the n^{th} and the $(n+1)^{th}$ cell.

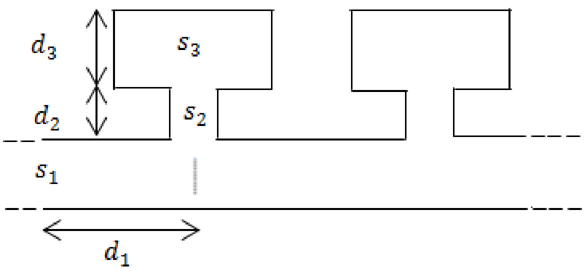

Figure1 Periodic structure with branched simple Helmholtz Resonator

$d_1$is the length of the unit cell and which is also the period of the periodic structure. $d = d_2 + d_3$is the length of the Helmholtz resonator.

2.1.*Transfer matrix*

In this section we present the formalism. We write thetransfer matrix for an infinite super lattice with a period of ‘n’ layers and calculate the band structure of the dispersionrelation of normal acoustic modes. We also study the reflectioncoefficient of the incident sound waves on the finiteand semi-infinite systems. The transfer matrix approach based on the one-dimensional plane wave theory isdescribed in detail by many authors for several acoustic elements and silencer configurations. In the absence of mean flow, we applied the plane wave theory. The effect of higher order modes, temperature gradients and viscous effect are excluded from the analytical procedure described below. These latter effects, which can be included in special cases, are not discussed in this section. We note T the transfer matrix of an elementary cell (fig.2) and we have:

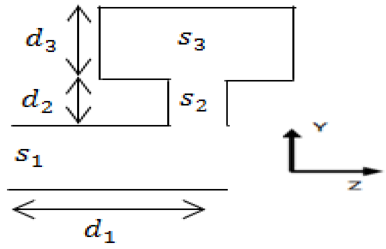

Figure 2 : Elementarycell

$$T_{cell} = \begin{pmatrix} A_{cell} & B_{cell} \\ C_{cell} & D_{cell} \end{pmatrix} = T_1 T_2 \qquad \text{eq.1}$$

Where T_1 is the transfer matrix corresponding to the element of the main guide of length "d_1" and T_2 the matrix representing the effect of the Helmholtz resonator. The effect of the connection of the Helmholtzresonator on the main guide can be represented by the following equivalent electric circuit:

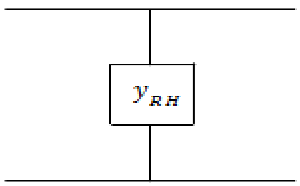

Figure3:Equivalent electric circuit of the Helmholtz Resonator (HR)

Thus, the matrices corresponding to each element of the unit cell are:

$$T_1 = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} cos(kd) & jz_{c\,1}\,sin(kd) \\ jy_{c\,1}\,sin(kd) & cos(kd) \end{pmatrix} \text{and} T_2 = \begin{pmatrix} 1 & 0 \\ Y_{HR} & 1 \end{pmatrix} \qquad \text{eq.2}$$

$T_1$is the transfer matrix between two elementary cell (nth and the *(n+1)*th cell) and T_2 represent the effect of the branched HR.Z_{HR} is the equivalent acoustic impedance of a Helmholtzresonator, k=ω/cis the wave number, ρ the air densityand c the speed of sound. $z_{c\,1} = \frac{\rho c}{s_1} = 1/y_{c\,1}$is the acoustic impedance characteristic of the main guidewith cross section s_1. We obtain

$$T_{cell} = \begin{pmatrix} cos(kd) + jz_{c\,1}Y_{HR}\,sin(kd) & jz_{c\,1}\,sin(kd) \\ jY_{HR}\,sin(kd) + jy_{c\,1}cos(kd) & cos(kd) \end{pmatrix} \qquad \text{eq.3}$$

2.1.*Evaluation of the equivalent impedance $Z_{HR}$to the resonator*

Acoustic quantitiesat the input and the output of the resonator are connected with the transfer matrix T_{HR}:

$$\begin{pmatrix} p_0 \\ u_0 \end{pmatrix} = T_{HR} \begin{pmatrix} p_d \\ u_d \end{pmatrix} = \begin{pmatrix} A_{HR} & B_{HR} \\ C_{HR} & D_{HR} \end{pmatrix} \begin{pmatrix} p_d \\ u_d \end{pmatrix} \qquad \text{eq.4}$$

where p_0 and u_0 are the pressure and volume velocity at the input of resonator.p_d and u_d are the pressure and volume velocity at the output of resonator. We obtain:

$$A_{HR} = cos(kd_2)\, cos(kd_3) - z_{c2}\, y_{c3}\, sin(kd_2)\, sin(kd_3) \quad \text{eq.5a}$$

$$B_{HR} = j\, y_{c3}\, cos(kd_2)\, sin(kd_3) + j\, z_{c3}\, sin(kd_2)\, cos(kd_3) \quad \text{eq.5b}$$

$$C_{HR} = j\, y_{c2}\, sin(kd_2)\, cos(kd_3) + j\, y_{c3}\, cos(kd_2)\, sin(kd_3) \quad \text{eq.5c}$$

$$D_{HR} = z_{c3} y_{c2}\; sin(kd_2)\, sin(kd_3) + cos(kd_2)\, cos(kd_3) \quad \text{eq.5d}$$

$z_{ci} = \frac{\rho c}{s_i} = 1/y_{ci}$ (i=2, 3).$z_{c2}$is the characteristic impedance of the Helmholtz resonator neck and z_{c3} the characteristic impedance corresponding to the cavity having the cross section of S_3 .
The termination of the resonator is rigid, so we have, $u_d = 0$ and

z_{HR} is:$$z_{HR} = \frac{A_{HR}}{C_{HR}} = \frac{\cos(kd_2)\cos(kd_3) - z_{c2}\, y_{c3} \sin(kd_2)\sin(kd_3)}{j\, y_{c2} \sin(kd_2)\cos(kd_3) + j\, y_{c3} \cos(kd_2)\sin(kd_3)} \quad \text{eq.6}$$

From where

$$z_{HR} = j\frac{z_{c2}\tan(kd_3) - z_{c3}\,\mathrm{cotan}(kd_2)}{z_{c3}\, y_{c2} + \mathrm{cotan}\,(kd_2)\tan(kd_3)} = 1/y_{HR} \quad \text{eq.7}$$

2.1.1*The dispersion relation*

The dispersion relation is given by a classical known equation:

$$\cos(qd_1) = \frac{1}{2}\, Tr\,(T_{cel}) = \frac{1}{2}\,(A_{cel} + D_{cel}) \quad \text{eq.8}$$

Where $Tr\,(T_{cel})$is the trace of the matrix T_{cel} and q is the Bloch's wave number. This equation is satisfiedonly for certain values of q. This means that the corresponding dispersion relation will present a band structure analogous to those of electron in a crystal and light in superlattice.

Charles **E.** Bradley [10] explains (see fig4) in the general case how varies *Re(qd*$_1$*)* and *Im(qd*$_1$*)*according to the frequency.

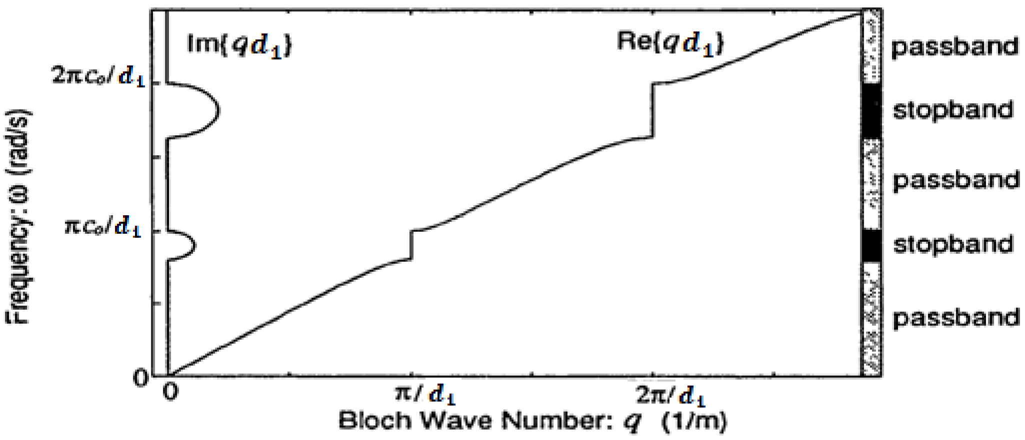

Figure 4: Illustration of stop-band and pass-band in general periodic structure [10]

Finally, the transfer matrix corresponding to one elementary cell is:

$$T_{cel} = \begin{pmatrix} A_{cell} & B_{cell} \\ C_{cell} & D_{cell} \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ y_{HR} & 1 \end{pmatrix} = \begin{pmatrix} A_1 + y_{HR} B_1 & B_1 \\ C_1 + y_{HR} D_1 & D_1 \end{pmatrix} \quad \text{eq.9}$$

We deduce the following value of dispersion relation:

$$cos(qd_1) = cos(kd_1) + \frac{1}{2}\, j\, z_{c1} y_{HR}\, sin(kd_1) \quad \text{eq.10}$$

This relationship is very important; it allows us to define the permitted bands and band gaps of the studied periodic structure. There are two cases:

- When $\cos(qd_1) < 1$, then q is real and Bloch waves propagate through the structure without loss, corresponding to the pass-band

- When $\cos(qd_1) > 1$, thenq is complex and Bloch waves are evanescent therefore these frequencies correspond to the band gap.

The cutoff frequencies are given by the following relationship:$\cos(qd_1) = 1$.

The eq.10may be calculated by another approach using the propertiesof continuity of both acoustics and volume velocity at each junction ofHR (see for example Y. Zhang [11]). This relation will be used to calculate the acoustic band structure of the infinite systems. While the real part of the Bloch wave number (Re(q)) is commonly used in the literatures to characterize the phase change of propagating waves in a pass band; the imaginary part (Im(q)) can be employed to describe the amplitude decay of evanescent waves in a band gap.

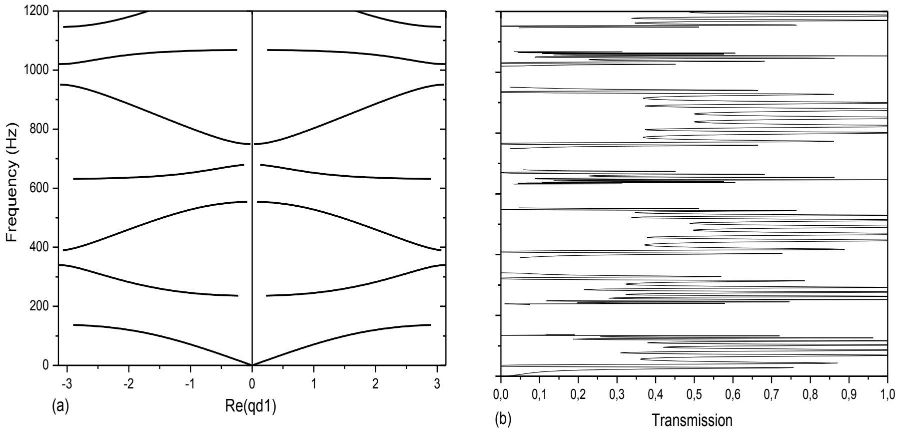

Figure 5: (a) Band structures and (b) Transmission T of a periodic system of 10 cells (N_c=10) and for d_1= 0.5m d_2=0.1m andd_3=0.3m

In this curve, we see alternatingpass-bands and bands gap. Low frequencies are the first permitted which shows that the filter acts as a low pass filter. The center of the first gap corresponding approximately to the special frequency f = 200Hz.

2.1.2*The transmission loss coefficient (TL)*

In applying the transfer matrix method, the transmission loss is obtained by a classical equation as:

$$\mathrm{TL}=20\log_{10}\left|\frac{1}{2}(\mathrm{A_{cell}} + \mathrm{y_{c\,1}}\,\mathrm{B_{cell}} + \mathrm{z_{c\,1}}\,\mathrm{C_{cell}} + \mathrm{D_{cell}})\right| \quad . \qquad \text{eq.11}$$

After a few calculi we obtain:

$$\mathrm{TL} = 20\log\left|1 + \frac{1}{2}\,\mathrm{z_{c1}}\,\mathrm{y_{HR}}\right| \qquad \text{eq.12}$$

This relationship is given by different authors see for example, Zhang [11] and Xu [12].

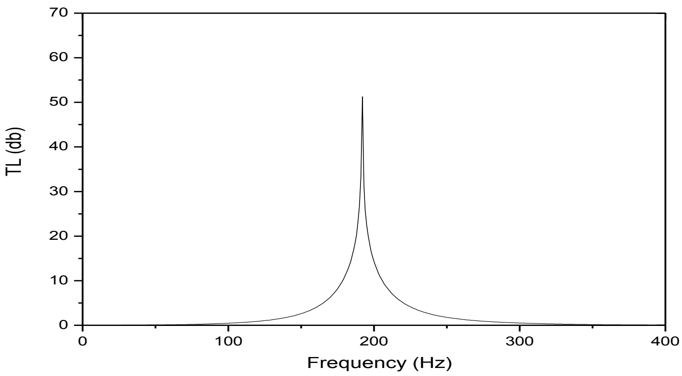

Figure 6: TL Coefficient corresponding to one cell (d_1= 0.5 m d_2=0.1m and d_3=0.3m)

This curve shows that the structure is selective and we note that the max ofTL corresponds approximately to the selectedfrequency (f=200Hz).

2.1.2*Conclusion*

We have presented the 2 x 2 transfer matrix method to study the response of HR periodic systems. We have obtained both the analytical dispersion relation and the transmission loss coefficient. This method is very simple and gives exact solutions for the problem. The max of TL corresponds approximately to the center of the first gap.

2.2.*Response function method or the Green's method*

In this section, we treat the problem with another method called "interface response theory (or Green function method) ".This method was developed in the beginning by Dobrzynski [1] for any composite. The main objective of this method is to obtain the inverse of the Green function for a given problem.We need the inverse of the Green function of the main guide and the inverse of the Green function of the HR. In the appendix we give the Green function corresponding to the general case of finite, semi-infinite and infinitestructures.
The main guide is considered as an infinite structure.So, we have:

$$g_1^{-1}(M,M) = \begin{pmatrix} a_1 & b_1 \\ b_1 & a_1 \end{pmatrix} = \begin{pmatrix} g_1^{-1}(0,0) & g_1^{-1}(0,d_1) \\ g_1^{-1}(d_1,0) & g_1^{-1}(d_1,d_1) \end{pmatrix} \qquad \text{eq.14}$$

This function represents the inverse of the Green function of the main guide. For the HR, we have:

$$g_{HR}^{-1}(M,M) = \begin{pmatrix} g_{HR}^{-1}(0,0) & g_{HR}^{-1}(0,d \ ) \\ g_{HR}^{-1}(d \ ,0) & g_{HR}^{-1}(d \ ,d \ ) \end{pmatrix} \qquad \text{eq.15}$$

This function corresponds to the inverse of the Green function of the connected resonator,it depends on the boundary conditions.
The domain of interfaces is reduced to a single interface M = {0} (the effect of each resonator on the main guide is reduced to a single point).
The inverse of the Green functions corresponding to the unit cell is given by:

$$g^{-1}(0,0) = g_1^{-1}(0,0) + N g_{HR}^{-1}(0,0) \qquad \text{eq.16}$$

with$g_1^{-1}(0,0) = a_1$ eq.17

On the other hand $g_{HR}^{-1}(0,0)$ is obtained after considering the conditions necessary limits.
Finally the inverse Green functions of the elementary cell in the interface space M = {0} is given by:

$$g^{-1}\ (M,M) = \begin{pmatrix} a_1 + N g_{HR}^{-1}(0,0) & b_1 \\ b_1 & a_1 \end{pmatrix} \qquad \text{eq.18}$$

where [1,3,17]:$a_1 = -\frac{F_1 C_1}{S_1}$, $b_1 = \frac{F_1}{S_1}$ and we put $C_1 = \mathrm{ch}\,kd_1$, $S_1 = sh\,kd_1$ and $F_1 = -j\frac{\omega}{Z_{c1}}$.
We note thatS$_i$ notation may cause ambiguity with the lowercase letter s$_i$ that means a cross sectional area of waveguide.
$g_{HR}^{-1}(0,0)$is the inverse of the Green function corresponding to the HR and is valid for all kinds of resonators connected whatever their forms.
The Green function of the overall system [1,3,17] is then obtained as an infinite trigonal matrix formed by the superposition of elements $g_i^{-1}(0,0)$ defined in the interface domain consisting of all junction i (or interface i).

$$g^{-1}(M,M) = \begin{pmatrix} A & b_1 & & & & \\ b_1 & B & b_1 & & & \\ & b_1 & B & b_1 & & \\ & & b_1 & B & b_1 & \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \qquad \text{eq.19}$$

with:$A = a_1 + N g_{HR}^{-1}(0,0)$, $B = 2a_1 + N g_{HR}^{-1}(0,0)$.
N is the number of identical connected resonator in each junction. For the rest we take a single resonator in each cell(N=1).

We note that this matrix is equivalent to a dynamic matrix of a monoatomic linear chain with a single spring constant [1].By analogy with the monatomic linear chain [1], the dispersion relation of a connected array of resonators can be given by:

$$2a_1 + g_{HR}^{-1}(0,0) + 2b_1\cos(qd_1) = 0 \qquad \text{eq.20}$$

from where

$$\cos(qd_1) = \cos(kd_1) - \frac{1}{2}\frac{S_1}{F_1}g_{HR}^{-1}(0,0) \qquad \text{eq.21}$$

This is the general equation of dispersion corresponding to all forms of connected resonators. Whatever the shape of the connected resonator (HR or other), the determination of the element $g_{HR}^{-1}(0,0)$ of the inverse matrix of the resonator connected Green function provides the dispersion relation of the system.

The transmission coefficient is given by:

$$t = -2F_1 g(0,0) \qquad \text{eq.22}$$

The power transmission is given by:$T = |t|^2$

2.2.2.*Green inverse function.Dispersion relation and transmission loss coefficient*

The inverse of the Green function of HR is:

$$g_{HR}^{-1}(M,M) = \begin{pmatrix} a_2 & b_2 & 0 \\ 0 & a_2 + a_3 & b_3 \\ 0 & b_3 & a_3 + F_4 \end{pmatrix} \qquad \text{eq.23}$$

As the resonator includes three interfaces then this matrix is (3x3) order. F_4 represents termination of the HR. The inverse of the Green function corresponding to the unit cell [18] can be obtained as the sum of two green functions. Thus, the domain of interfaces that reduced to a single interface M = {0} represents the place of anointing of HR with the main waveguide (we have the same equations; eq.16 and 17).

This is the matrix element corresponding to the inverse Green function of the finite guide.

The wall of the resonator (cavity) is rigid, then the acoustic flow vanishes (u = 0) therefore the quantity F_4 vanishes ($F_4 = 0$).

After a few calculi we obtain:

$$g_{\mathrm{HR}}(0,0) = \frac{a_3(a_2+a_3) - b_3^2}{\det\left[g_{\mathrm{HR}}^{-1}(M,M)\right]} \qquad \text{eq.24}$$

where:

$$\det[g_{\mathrm{HR}}^{-1}(M,M)] = F_2^2 F_3 \quad \frac{C_3}{S_3} + F_2\ F_3^2\frac{C_2}{S_2} \qquad \text{eq.25}$$

Finally, we have:

$$g_{\mathrm{HR}}(0,0) = \frac{F_2\ F_3\ C_2\ C_3\ -F_3^2 S_2\ S_3}{F_2^2 F_3\ C_3\ S_2\ \ +F_2\ F_3^2 C_2\ S_3} \qquad \text{eq.26}$$

After making the necessary substitutions, we find the following expression:

$$g_{\mathrm{HR}}{}^{-1}(0,0) = -\omega\frac{\mathrm{z_{c3}\,yc_2} + \ \mathrm{cotan}\ (k\mathrm{d_2})\tan(k\mathrm{d_3})}{\mathrm{z_{c2}}\ \tan(k\mathrm{d_3}) - \mathrm{z_{c3}}\ \mathrm{cotan}\ (k\mathrm{d_2})} \qquad \text{eq.27}$$

The dispersion relation of a simple HR network may be given by:

$$\cos(qd_1) = \cos(kd_1) - \frac{1}{2}\frac{z_{c1}}{\omega}\sin(kd_1)\, g_{HR}^{-1}(0,0) \qquad \text{eq.28}$$

This equation leads to the same value as given by eq.10 given by the method of transfer matrix.We have also:

$$g^{-1}(0,0) = -2F_1 + g_{\mathrm{HR}}{}^{-1}(0,0)$$

and this leads to:

$$g(0,0) = \frac{-1}{2F_1 - g_{\mathrm{HR}}{}^{-1}(0,0)}\ .$$

We deduce the transmission coefficient from eq.22:

$$t = \frac{1}{1 - j\,\frac{1 z_{c1}}{2\omega} g_{\mathrm{HR}}{}^{-1}(0,0)}$$ eq.29

The power transmission T may be deduced easily by: $T = |t|^2$ and we find the TL value obtained by the method of the transfer matrix by writing: $TL = 10 \log\left|{}^{1}/_{T}\right|$ (see eq.12).

2.2.3. Conclusion

This study shows that the method of the interface response function leads to the same results as those obtained by the method of the transfer matrix.We have tested this method for other periodic structures like the identical expansion chambers network.This formalism seems complicated but gives simple results. The only difficulty is the calculation of the inverse of the Green function.

Conclusions

This study shows good agreement between the two methods used to know the method of the transfer matrix method and the response of the interface function.We obtained similar results for the dispersion relation and the coefficient of transmission losses.The goal was the confrontation between the two methods; the numerical part was intentionally not fully exposed. The perspective of this work is to make a detailed numerical analysis and to examine the case where it exist a defect in the periodic structure.

Annex: Application of the method (RIF) for different waveguide

Green's functions of elementary waveguide

1. *Green's function of an infinite waveguide*

We consider an infinite waveguide taken in Cartesian coordinates (o, x, y, z). It was established [13, 14, 15] that the Fourier transform of the Green's function between two points X (x, y, z) and X' (x', y', z') is given by :

$$G(z, z') = -\frac{1}{2F} e^{-jk|z - z'|}$$ eq.1

with: $F = -j\,\frac{\omega}{Z_c}$. eq.2

$z_c = \frac{\rho c}{S} = 1/y_c$ is the acoustic impedance characteristic of the structure and c the speed of sound.

2. *Green's function of a semi-infinite waveguide*

When we have a semi-infinite waveguide in the half space x≥0 Fig.1'

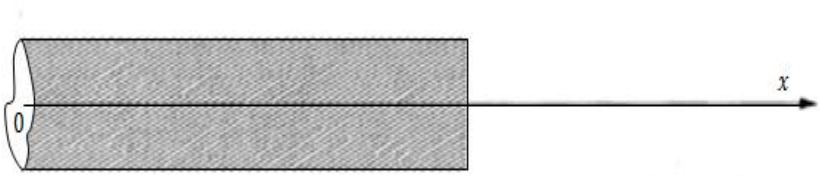

Figure 1':Semi-infinite waveguide

The inverse of the Green function at the surface x = 0 is given by [16]

$$g^{-1}(M, M) = g^{-1}(0,0) = -F$$ eq.3

3. *Green's function of a finite waveguide*

For a structure having a finite thickness d (Fig.2')

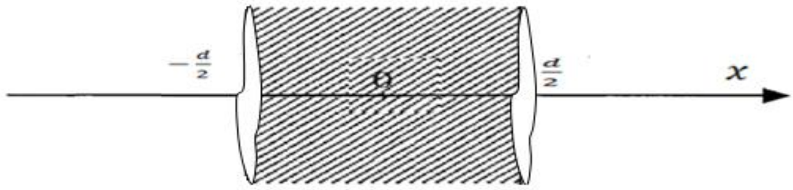

Figure 2': Finite waveguide

The space of the interfaces is formed on both surfaces located at x = -d / 2 and x = d / 2. The inverse of the Green function is given by [18]:

$$g^{-1}(M,M) = \begin{pmatrix} -\frac{F\ \ S_i}{C_i} & \frac{F}{S_i} \\ \frac{F}{S_i} & -\frac{F\ \ S_i}{C_i} \end{pmatrix} \qquad \text{eq.4}$$

with: $F - j\frac{\omega}{z_c}; C_i = \cos kd, S_i = \ -j\sin kd\ \ , k = \frac{\omega}{c}$ the wave number.

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