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Computation of the New Retaining Wall Consisted of Cylindrical Thin Shell and Flat Slabs by the Classical Method

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Abstract
In this article, to reduce the weight of the concrete gravity walls, a new structure has been proposed. In order to achieve this purpose, cylindrical thin shells are adapted with concrete gravity walls. Then to calculate new retaining wall, a new method, based on the principles of mathematics and engineering mechanics, is presented. In solving the new retaining wall, Fourier series and shells theory and the classical analysis method are used. So, an example and the numerical results in this regard will be discussed.

Keywords: Thin shells, Gravity walls, New retaining wall, Cylindrical shells

1. Introduction
In various projects such as road, bridge building, landscaping, retaining walls are used. In general, different kinds of retaining walls are used to stop the soil from falling everywhere, i.e. there is a need for lateral support in the case of upright excavation. The extensive use of the retaining walls has created a new field of study. Therefore, in this article cylindrical thin shells are adapted with concrete gravity walls and also a new structure is proposed. In other words, the calculation of lightened concrete gravity walls consisted of cylindrical thin shells and flat slabs, are discussed through the mathematical and classical method in this paper.
The theory of plates and shells is expressed in professional books and articles in great details. In this article, a number of selected resources are given in the references [1-6].
The results of studies on the cantilever retaining walls, counterfort and buttress retaining walls and gravity walls composed of three flat slabs, multi cylindrical shells and the finite element method by the author are found in numerous published articles [7-19]. However, studies on the gravity walls consisted of cylindrical thin shells and the mathematical and classical methods by the author for the first time is presented in this paper.

2. Materials and methods
2.1. *Calculation of the new retaining wall*
Attention to the generalities of structure and the selection of its geometrical shape and the relationship of the main shell with the other elements are some of the important considerations in shell structure designing. They are important because of their behavior, balance and general stability of the structure; specifically, the performance of structure against forces. These points are also considered in the proposed structure for a retaining wall. As Figure 1 illustrates, if the internal space of concrete gravity walls is considered to be hollow, and the cases of concrete gravity walls are supposed to be cylindrical thin shells and flat slabs, and to increase its stability the hollow spaces are filled with soil, the resulting retaining walls will be considerably economical regarding consumption of material.

In other words, cylindrical shells for the front bearing element of the retaining wall, a flat slab for the base element, and a rectangular slab for the upper bearing element are taken into consideration.

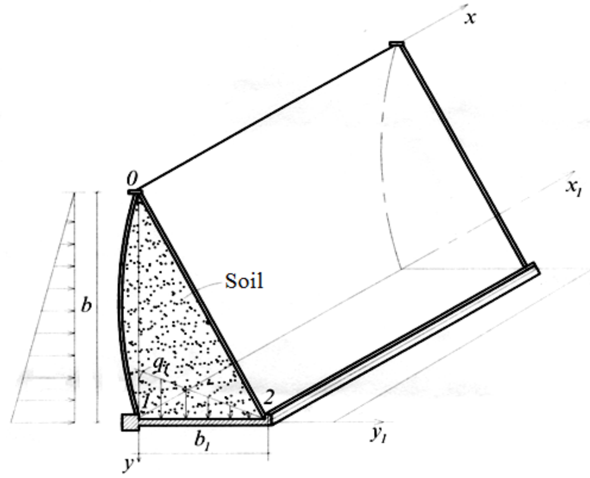

Figure 1: Shell retaining wall.

According to soil lateral force theories, the wall shell will be influenced by a triangular load resulting from soil active pressure. Also, general proportions of the proposed retaining wall are regarded as being the same as the gravity retaining wall. The connection of the wall shell with the base slab is assumed rigid. Also the base slab is considered far thicker than cylindrical shells. Further the rigidity of base slab flexure is considered much. The linkage place of cylindrical shell with base slab against torsion has much rigidity. Considering all these factors, the linkage place of base slab with cylindrical shells is rigid and restrained. The rigidity of linkage place of base slab with cylindrical shells leads the computation of retaining wall to two independent problems. The conditions of rigid (restrained) linkage are presented as follows. If for cylindrical shells $y=b$ then we will have:

$$w=0,\ \frac{\partial w}{\partial y}=0,\ \ v=0,\ \ \varepsilon_x=0 \tag{1}$$

In $y=0$ is considered hinge linkage and boundary condition is regarded as follows:

$$w=0,\ M_y=0,\ N_y=0,\ \varepsilon_x=0 \tag{2}$$

For base slab the linkage condition is as follows:

$$(y_1=0)\qquad w_1=0,\ \frac{\partial w_1}{\partial y}=0 \tag{3}$$

$$(y_1=b_1)\qquad w_1=0,\ M_{y_1}=0 \tag{4}$$

In place $X=0$ & $X=a$, linkages are supposed hinge type.

$$w=0,\ M_x=0,\ N_x=0,\ \varepsilon_y=0 \tag{5}$$

Therefore the computation of elements of retaining wall by applying the following differential equations and considering border and linkage condition is done. We have for front cylindrical shells:

$$D\Delta^2 w - \frac{1}{R}\frac{\partial^2 \varphi}{\partial x^2} = q_0 \cdot \frac{y}{b} \tag{6}$$

$$\frac{1}{Eh}\Delta^2 \varphi - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0 \tag{7}$$

For base slab:

$$D_1 \Delta^2 w_1 + k w_1 + k_s \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) = q(y) \tag{8}$$

The solving of main differential equation of the bending theory of cylindrical shells by considering accepted border conditions is selected as follows:

$$w = \sum_m \sum_n B_{mn} \sin \lambda_m x \sin \mu_n y - \frac{1}{D}\overline{F}(y) \sum_m N_m \sin \lambda_m x \tag{9}$$

$$\varphi = \sum_m \sum_n A_{mn} \sin \lambda_m x \sin \mu_n y - \sum_m \left[\frac{1}{\lambda_m^2} . y_b - \nu \overline{F}(y) \right] \xi_m \sin \lambda_m x \tag{10}$$

In order to make computations simple, the problem solving is presented in this way:

$$B_{mn} = - \frac{8(-1)^n (m^2 + \gamma^2 n^2)^2}{mn\pi^6 D^*(m,n)} . \frac{q_0 a^4}{D} + (-1)^n E_2(m,n) N_m - (-1)^n E_3(m,n) \xi_m \tag{11}$$

$$A_{mn} = - \frac{96(1-\nu^2)^n . m . (-1)^n}{n\pi^8 D^*(m,n)} . \frac{a^2}{h^2} . \frac{q_0 a^4}{D} - (-1)^n E_{22}(m,n) N_m - (-1)^n E_{33}(m,n) \xi_m \tag{12}$$

The marks and symbols used in this calculation are as follows:

$$D^*(m,n) = (m^2 + \gamma^2 n^2)^4 + \frac{12(1-\nu^2)}{\pi^4} . \frac{a^2}{h^2} . \frac{a^2}{R^2} \tag{13}$$

$$E_2(m,n) = \frac{2m^2 (m^2 + \gamma^2 n^2)}{n\pi^3 D^*(m,n)} \left[2 + \frac{m^2}{\gamma^2 n^2} + \frac{12(1-\nu^2)}{\pi^4} . \frac{m^2}{\gamma^2 n^2 \overline{\Delta}_{mn}^2} . \frac{a^2}{h^2} . \frac{a^2}{R^2} \right] \frac{a^2}{D} \tag{14}$$

$$E_3(m,n) = \frac{2(m^2 + \gamma^2 n^2)^2}{n\pi^5 D^*(m,n)} \left[1 + \nu \frac{m^2}{\gamma^2 n^2} - \frac{m^4}{\gamma^2 n^2} - \frac{m^4}{(m^2 + \gamma^2 n^2)^2} \left(1 + \nu \frac{m^2}{\gamma^2 n^2} + 2\nu \right) \right] \frac{a^4}{RD} \tag{15}$$

$$E_{22}(m,n) = \frac{24 m^2 (1-\nu^2) \overline{\Delta}_{mn}^2}{n\pi^5 D^*(m,n)} \left[\frac{m^2}{n^2 \gamma^2} - \frac{m^4}{\overline{\Delta}_{mn}^2} \left(2 + \frac{m^2}{n^2 \gamma^2} \right) \right] \frac{a^4}{Rh^2} \tag{16}$$

$$E_{33}(m,n) = \frac{2m^2\left(m^2+\gamma^2n^2\right)^2}{n\pi^3 D^*(m,n)}\left[1+\nu\frac{m^2}{\gamma^2n^2}+2\nu+\frac{12\left(1-\nu^2\right)}{\pi^4}\cdot\frac{1}{\overline{\Delta}_{mn}^2}\cdot\frac{a^4}{R^2h^2}\left(1+\nu\frac{m^2}{n^2\gamma^2}+2\nu\right)\right]a^2 \quad (17)$$

$$\overline{\Delta}_{mn}^2 = \left(m^2+\gamma^2n^2\right)^2 \quad (18)$$

The border condition of rigid linkage of base slab with cylindrical shells is paid attention. To reach this purpose, boundary conditions of rigid linkage of base slab with cylindrical shell such as flexure & stress functions is determined as follows:

$$\left.\frac{\partial w}{\partial y}\right|_{y=b} = 0,\ \left.\frac{\partial^3\varphi}{\partial y^3}+(2+\nu)\frac{\partial^3\varphi}{\partial x^2\partial y}\right|_{y=b} = 0 \quad (19)$$

Other conditions such as $w=0$ and $\varepsilon_x = 0$ is supplied. Stress & flexure functions existing in solving problem are placed in the above conditions and after a series of mathematical operations the following algebraic equation system is resulted.

$$-\frac{b^2}{3D}N_m = -\sum_n n\pi B_{mn}.(-1)^n \quad (20)$$

$$\left[-2(1+\nu)\gamma+\frac{\nu(2+\nu)}{3\gamma}m^2\pi^2\right]a^2\xi_m = -\sum_m(-1)^n\Delta_{mn}^{(2+\nu)}\pi^3A_{mn} \quad (21)$$

$$\Delta_{mn}^{(2+\nu)} = n^3\gamma^3+(2+\nu)m^2n\gamma \quad (22)$$

After including two dimensional Fourier series coefficients (A_{mn} & B_{mn}) in the mentioned algebraic equation system, binominal algebraic equations are resulted:

$$a_1(m)\frac{a^2}{D}N_m+b_1(m)\frac{a^2}{RD}\xi_m = \delta_1(m).\frac{q_0a^4}{D} \quad (23)$$

$$a_2(m)\frac{a^4}{Rh^2}N_m+b_2(m).a^2\xi_m = \delta_2(m).\frac{q_0a^4}{R}$$

The coefficients of this algebraic equation system are presented as follows:

$$a_1(m) = -\frac{1}{3\gamma^2}-\sum_n(-1)^n n\pi E_2(m,n) \quad (24)$$

$$b_1(m) = \sum_n(-1)^n n\pi E_3(m,n) \quad (25)$$

$$\delta_1(m) = \frac{8}{m\pi^5}-\sum_n\frac{\left(m^2+n^2\gamma^2\right)^2}{D^*(m,n)} \quad (26)$$

$$a_2(m) = -\sum_n \pi^3 \Delta_{mn}^{(2+\nu)} E_{22}(m,n) \tag{27}$$

$$b_2(m) = -2(1+\nu)\gamma \frac{\nu(2+\nu)}{3\gamma^2} m^2\pi^2 - \sum_n \pi^3 \Delta_{mn}^{(2+\nu)} n\pi E_{33}(m,n) \tag{28}$$

$$\delta_2(m) = \frac{96(1-\nu^2)}{\pi^5} \frac{a^2}{h^2} \sum_n \frac{m}{n} \frac{\Delta_{mn}^{(2+\nu)}}{D^*(m,n)} \tag{29}$$

$$\Delta_{mn}^{(2+\nu)} = n^3\gamma^3 + (2+\nu)m^2 n\gamma \tag{30}$$

One dimensional series coefficients of stress and flexure functions in solving algebraic equation system, i.e. N_m & ξ_m is determined as follows:

$$N_m = \frac{1}{\Delta^*(m,n)}[\delta_1.(m).b_2.(m) - \frac{a^2}{R^2}\delta_2.(m).b_1.(m)].q_0 a^2 \tag{31}$$

$$\xi_m = \frac{1}{\Delta^*(m,n)}[\delta_2.(m).a_1.(m) - \delta_1.(m).a_2.(m)\frac{a^2}{h^2}].\frac{q_0 a^2}{R} \tag{32}$$

Here $\Delta^*(m, n)$ determinant is formed of algebraic equation coefficients:

$$\Delta^*(m,n) = a_1(m).b_2(m) - \frac{a^2}{h^2}.\frac{a^2}{R^2} a_2(m).b_1(m) \tag{33}$$

After determining the coefficients of the one-dimensional series in solving the problem by using directions 12 and 19, the coefficients of the two-dimensional series are easily calculated. So the problem of combining base slab with cylindrical shells is solved, and place changes and internal forces created in cylindrical shells are calculated by using specific instructions.

3. Results and discussion

3.1. *Numerical results of solving problem*

Numerical examples of solving the problem by choosing numerical values for physical and geometrical parameters of cylindrical shells are made **(**Figure 2). Computations are represented for the following geometrical quantities:

$$b = R, \quad f = \frac{b^2}{8R} = \frac{b}{8} \tag{34}$$

In case cylindrical shells are thin, the shells thickness is determined as follows:

$$h_{\min} \le \frac{1}{30}R, \quad \frac{R}{h} = \frac{b}{h} = 60 \tag{35}$$

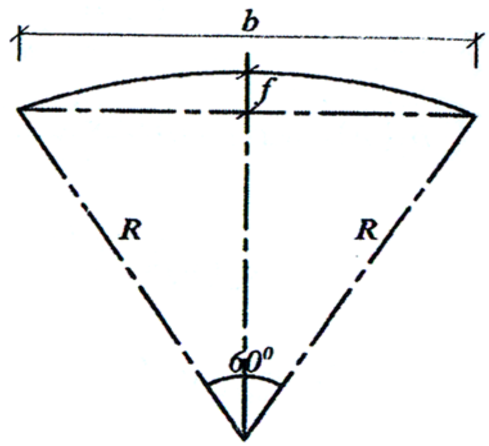

Figure 2: Geometrical parameters of cylindrical shell.

By considering the proportion of cylindrical shell in plan ($\gamma = \frac{a}{b}$), the amount of $\gamma = 1.5$ are included in computations. The height coefficient of cylindrical shells is determined as follows:

$$\psi = \frac{f}{h} = \frac{b}{8h} = \frac{60}{8} = 7.5 \tag{36}$$

Based on geometrical parameters accepted above, the coefficient included in the problem solving is computed as presented in the Tables 1 to 9:

Table 1: Amount of $\overline{\Delta}_{mn}^{2}$.

m / n	1	2	3	4	5
1	10.5625	100.0	451.5625	1369	3277.5625
3	126.5625	324.0	855.5625	2025	4257.5625
5	742.5625	1156.0	2047.5625	3721	6601.5625

Table 2: Amount of $\Delta_{mn}^{(2+\nu)}$.

m / n	1	2	3	4	5
1	6.6300	33.51	100.89	229.02	438.15
3	32.670	85.59	179.01	333.18	483.15
5	84.750	189.75	335.25	541.50	828.75

Table 3: Amount of $D^*(m,n)$.

m / n	1	2	3	4	5
1	2292.066	12180.50	206089.1	1876341.5	10744596
3	192638.6	281596.5	908607.70	4277245.5	18303458
5	1914211.6	2699148	5555324.4	15208653.5	44943439

Table 4: Amount of $B_{mn}^{(0)}$.

m / n	1	2	3	4	5
1	3.8351×10^{-5}	-3.4161×10^{-5}	6.0781×10^{-6}	-1.5179×10^{-6}	5.0771×10^{-7}
3	1.8225×10^{-6}	-1.5955×10^{-6}	8.7068×10^{-7}	-3.2833×10^{-7}	1.2905×10^{-7}
5	6.4565×10^{-7}	-5.9403×10^{-7}	2.0448×10^{-7}	-1.0180×10^{-7}	4.8895×10^{-8}

Table 5: Amount of $A_{mn}^{(0)}$.

m / n	1	2	3	4	5
1	0.034720	-3.2669×10^{-3}	1.2871×10^{-4}	-1.0603×10^{-3}	1.4813×10^{-6}
3	0.001239	-4.2391×10^{-4}	8.7585×10^{-5}	-1.3954×10^{-5}	2.6087×10^{-6}
5	2.0787×10^{-4}	-7.3709×10^{-5}	2.3956×10^{-5}	-6.5408×10^{-6}	1.7707×10^{-6}

Table 6: Amount of $E_2(m,n)$.

m / n	1	2	3	4	5
1	0.02799	0.00854	0.0001075	0.2438×10^{-4}	0.7973×10^{-5}
3	0.02858	0.00325	0.0001731	0.8316×10^{-4}	0.6055×10^{-4}
5	0.02861	0.00346	0.0009012	0.0003056	0.0001227

Table 7: Amount of $E_3(m,n)$.

m / n	1	2	3	4	5
1	0.2837×10^{-4}	0.2710×10^{-4}	0.4796×10^{-5}	0.1192×10^{-5}	0.3986×10^{-6}
3	0.1663×10^{-5}	0.2980×10^{-5}	0.1930×10^{-5}	0.7637×10^{-6}	0.3020×10^{-6}
5	0.6705×10^{-5}	0.6891×10^{-5}	0.5915×10^{-6}	0.3492×10^{-6}	0.1808×10^{-6}

Table 8: Amount of $E_{22}(m,n)$.

m / n	1	2	3	4	5
1	0.7475×10^{-4}	0.2813×10^{-4}	0.2494×10^{-5}	0.3651×10^{-6}	0.7973×10^{-7}
3	0.8005×10^{-5}	0.1095×10^{-5}	0.5096×10^{-6}	0.1442×10^{-6}	0.4212×10^{-7}
5	0.2234×10^{-5}	0.3171×10^{-5}	0.2306×10^{-6}	0.1124×10^{-6}	0.4764×10^{-7}

Table 9: Amount of $E_{33}(m,n)$.

m / n	1	2	3	4	5
1	0.06640	0.006243	0.000293	0.3460×10^{-4}	0.7892×10^{-5}
3	0.01104	0.000602	0.000757	0.000172	0.5113×10^{-4}
5	0.00733	0.001582	0.000562	0.000230	0.8383×10^{-4}

In binominal equation system for m=1, 3, 5 the following quantities are calculated (Tables 10 and 11):

Table 10: Amount of $a_1(m)$, $b_1(m)$, $\delta_1(m)$.

m	$a_1(m)$	$b_1(m)$	$\delta_1(m)$
1	-0.08781	0.4458×10^{-4}	0.4194×10^{-3}
3	-0.07905	0.1627×10^{-6}	0.2806×10^{-4}
5	-0.07338	0.1820×10^{-4}	0.7473×10^{-5}

Table 11: Amount of $a_2(m)$, $b_2(m)$, $\delta_2(m)$.

m	$a_2(m)$	$b_2(m)$	$\delta_2(m)$
1	-0.0561	-24.099	4.0165
3	-0.02162	-15.752	0.5110
5	-0.03008	-23.698	0.2965

To solve problem proper amount of the above mentioned, the following amounts are resulted:

$$N_1 = -0.004090 q_0 a^2,\ N_3 = -0.000355 q_0 a^2,\ N_5 = -0.0001018 q_0 a^2$$

$$\xi_1 = -0.1666\frac{q_0 a^2}{R},\ \xi_3 = -0.03244\frac{q_0 a^2}{R},\ \xi_5 = -0.0125\frac{q_0 a^2}{R}$$

Therefore after one dimensional series coefficients of problem became obvious, the coefficients of two dimensional series are computed and the resulted amounts are presented in the Tables 12 and 13. It should be mentioned that one dimensional series, show faster convergence and its extension in practical computation is enough up to the trinomial.

Table 12: Amount of B_{mn}.

m / n	1	2	3	4	5	$N_m(qa^2)$
1	1.4222×10^{-4}	0.5893×10^{-4}	0.4719×10^{-5}	-1.1708×10^{-6}	3.9092×10^{-7}	-0.004090
3	-0.7953×10^{-9}	2.6098×10^{-6}	7.9230×10^{-7}	-3.2832×10^{-7}	1.2905×10^{-7}	-0.000355
5	3.3695×10^{-5}	7.5445×10^{-7}	2.4118×10^{-7}	-1.018×10^{-7}	4.3512×10^{-8}	-0.000102

Table 13: Amount of A_{mn}.

m / n	1	2	3	4	5	$\xi_m\left(\frac{qa^2}{R}\right)$
1	0.021180	-0.001284	0.2710×10^{-4}	-1.0415×10^{-5}	1.4183×10^{-6}	-0.1666
3	0.001015	-0.4088×10^{-3}	6.1746×10^{-5}	-1.3954×10^{-5}	1.6053×10^{-6}	-0.0322
5	2.0599×10^{-4}	-7.1091×10^{-5}	2.43765×10^{-5}	-6.5408×10^{-6}	4.7157×10^{-6}	-0.0125

So for geometrical amounts, stresses and deflection functions are considered and determined. Here we have paid attention to two cases: Considering the kind of cylindrical shells linkage to base slab in hinge or rigid form, in cylindrical shells, deflections, bending moment, and normal forces in shearing place x=0.5a is calculated:

$$w\big|_{x=0.5a} = (B_{11} - B_{31} + B_{51})\sin\frac{\pi y}{b} + (B_{12} - B_{32} + B_{52})\sin\frac{2\pi y}{b} + (B_{13} - B_{33} + B_{53})\sin\frac{3\pi y}{b} +$$
$$+ (B_{14} - B_{34} + B_{54})\sin\frac{4\pi y}{b} + (B_{15} - B_{35} + B_{55})\sin\frac{5\pi y}{b} - \frac{1}{D}\overline{F}(y)(N_1 - N_3 + N_5)$$

$$N_y\big|_{x=0.5a} = -\pi^2\Big[(A_{11} - 9A_{31} + 25A_{51})\sin\frac{\pi y}{b} + (A_{12} - 9A_{32} + 25A_{52})\sin\frac{2\pi y}{b} +$$
$$+ (A_{13} - 9A_{33} + A_{53})\sin\frac{3\pi y}{b} + (A_{14} - 9A_{34} + 25A_{54})\sin\frac{4\pi y}{b} + (A_{15} - 9A_{35} + 25A_{55})\sin\frac{5\pi y}{b}\Big] +$$
$$+ y_b(\xi_1 - \xi_3 + \xi_5) - \frac{\nu\pi^2}{6\gamma^2}(y_b^3 - y_b)(\xi_1 - 9\xi_3 + 25\xi_5)$$

$$M_y\big|_{x=0.5a} = \pi^2\Big\{\big[(1+\nu)B_{11} - (1+9\nu)B_{31} + (1+25\nu)B_{51}\big]\sin\frac{\pi y}{b} + \big[(4+\nu)B_{12} - (4+9\nu)B_{32} + (4+25\nu)B_{52}\big]\sin\frac{2\pi y}{b}$$
$$+\big[(9+\nu)B_{13} - 9(1+\nu)B_{33} + (9+25\nu)B_{53}\big]\sin\frac{3\pi y}{b} + \big[(16+\nu)B_{14} - (16+9\nu)B_{34} + (16+25\nu)B_{54}\big]\sin\frac{4\pi y}{b} +$$
$$+\big[(25+\nu)B_{15} - (25+9\nu)B_{35} + 25(1+\nu)B_{55}\big]\sin\frac{5\pi y}{b}\Big\} + y_b(N_1 - N_3 + N_5) - \frac{\nu\pi^2}{6\gamma^2}(y_b^3 - y_b)(N_1 - 9N_3 + 25N_5)$$

For the hinged linkage state between front cylindrical shells of retaining wall with base slab, the mentioned cases for deflection, bending moment, and normal forces are presented as follows:

$$w\big|_{x=0.5a} = 3.7174\times10^{-5}\sin\frac{\pi y}{b} - 3.3160\times10^{-5}\sin\frac{2\pi y}{b} + 5.2275\times10^{-6}\sin\frac{3\pi y}{b} +$$
$$-1.2914\times10^{-6}\sin\frac{4\pi y}{b} + 4.2756\times10^{-7}\sin\frac{5\pi y}{b},$$

$$M_y\big|_{x=0.5a} = 0.0004308\sin\frac{\pi y}{b} - 0.0013167\sin\frac{2\pi y}{b} + 0.0004863\sin\frac{3\pi y}{b} +$$
$$-0.0002058\sin\frac{4\pi y}{b} + 0.0001077\sin\frac{5\pi y}{b},$$

$$N_y\big|_{x=0.5a} = -0.2847\sin\frac{\pi y}{b} + 0.01277\sin\frac{2\pi y}{b} + 0.000598\sin\frac{3\pi y}{b} + 0.000479\sin\frac{4\pi y}{b} - 0.0002195\sin\frac{5\pi y}{b},$$

For the rigid linkage state between cylindrical shells and base slab, the aforementioned parameters can be computed in the same way:

$$w\big|_{x=0.5a} = 1.4559\times10^{-4}\sin\frac{\pi y}{b} - 0.5707\times10^{-4}\sin\frac{2\pi y}{b} + 0.4167\times10^{-6}\sin\frac{3\pi y}{b} - 0.9444\times10^{-6}\sin\frac{4\pi y}{b} + $$
$$+3.0538\times10^{-7}\sin\frac{5\pi y}{b} + 2.8422\times10^{-4}\left(y_b^3 - y_b\right),$$

$$M_y\big|_{x=0.5a} = 0.001817\sin\frac{\pi y}{b} - 0.002345\sin\frac{2\pi y}{b} + 0.0003763\sin\frac{3\pi y}{b} - 0.0001502\sin\frac{4\pi y}{b} + $$
$$+0.0000733\sin\frac{5\pi y}{b} - 0.003836 y_b + 0.000427\left(y_b^3 - y_b\right),$$

$$N_y\big|_{x=0.5a} = -0.1698\sin\frac{\pi y}{b} + 0.00511\sin\frac{2\pi y}{b} - 0.000646\sin\frac{3\pi y}{b} + 0.000477\sin\frac{4\pi y}{b} + $$
$$-0.000206\sin\frac{5\pi y}{b} - 0.1469 y_b + 0.0235\left(y_b^3 - y_b\right),$$

Based on this point, the amount of deflection, bending moment and normal forces created in shearing place *x=0.5a* is presented in the Table 14:

Table 14: Amount of deflection, bending moment, and normal forces created in shearing place *x=0.5a*.

y_b	For hinge linkage			For the rigid linkage		
	$10^{-4}w$	$10^{-3}M_y$	N_y	$10^{-4}w$	$10^{-3}M_y$	N_y
0.0	0	0	0	0	0	0
0.1	-0.03362	-0.3664	-0.0799	-0.1688	-1.3210	-0.0695
0.2	-0.0453	-0.7054	-0.1536	-0.2322	-1.7460	-0.1247
0.3	-0.00211	-0.7859	-0.2281	-0.1403	-1.9610	-0.1751
0.4	0.1268	-0.7689	-0.2587	-0.0941	-1.470	-0.2125
0.5	0.3176	-0.3773	-0.2833	-0.2804	-0.608	-0.2521
0.6	0.5166	0.9274	-0.2793	-0.5002	-0.471	-0.2886
0.7	0.6329	1.7989	-0.2299	-0.5190	-0.931	-0.2480
0.8	0.5848	1.7762	-0.1794	-0.3796	-0.331	-0.2174
0.9	0.3537	1.3301	-0.0950	-0.2981	-1.153	-0.1807
1.0	0.0	0.0	0.0	0.0	-3.832	-0.1469

In this way the forces resulted from lightened retaining walls are calculated.

The amount of internal forces for front shell from the lightened retaining wall (Figure 3):

$$N_d = \sigma_d \cdot 1 = \frac{M(y)}{W_x(y)} \tag{37}$$

Where:

$$W_x(y) = \frac{1 \cdot b^2(x)}{6}, \quad M(y) = \frac{q_o \cdot y^3}{6\,b}$$

$$N_d = \frac{q_o \cdot y^3}{b^2(y)}, \quad b(y) = \frac{y}{b} \cdot b_1$$

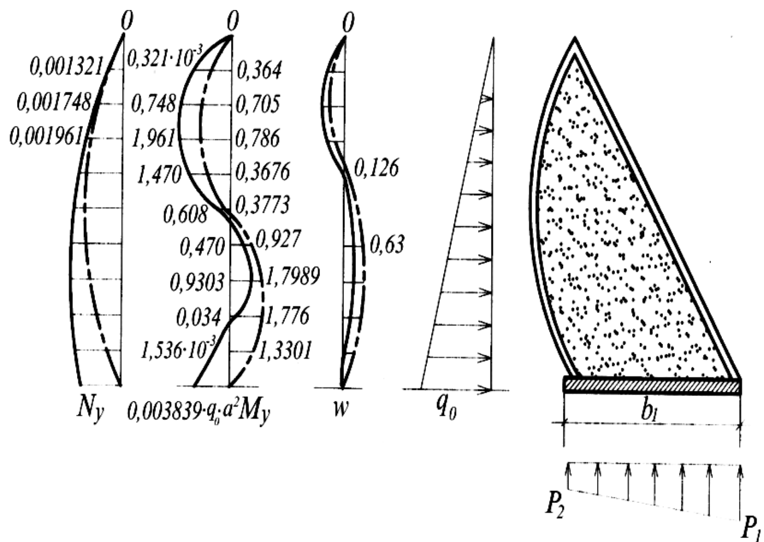

Figure 3: The amount of internal forces for front shell.

Conclusions

In this article is a new structure for the lightened gravity walls presented. Then to calculate new retaining wall, a new method is proposed. The research results presented in this article, for the first time bring about a classical method to calculate lightened gravity walls using the series and the mathematical principles. Also, for the first time cylindrical thin shells are adapted with concrete gravity walls in the proposed shape and form, and a numerical example is stated as well. Investigating the resulted amounts for the consumption concrete volume of the gravity retaining walls and lightened shell retaining wall, it can be seen that the use of shell retaining walls are considerably more economical in consumption of concrete and lead (about 90%) in comparison with concrete gravity retaining walls. To achieve this purpose, if the consumption concrete level used in gravity retaining wall is considered approximately $0.5b_1b$, this amount will be $0.04b_1b$ in shell retaining walls. (b_1 the width of wall section and b the wall height); so it is more economical to use the suggested retaining wall in this article.

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