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Investigation the Effects of Reynolds and Rayleigh Numbers on the Non-Newtonion Nanofluid (Al_2O_3/Propellant Dough) heat transfer

Samira Hashemi [1], Hossein Shokouhmand [2]

[1]*Master of Science, Department of Technical & Engineering, West Tehran Branch-Islamic Azad University, IRAN*
[2]*Professor, School of Mechanical Engineering, University of Tehran, Iran*

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Samira.hashemi.2011@gmail.com

Abstract
As plate heat exchangers have numerous applications and there is a lot of heat exchange between solid surfaces and fluids in different industries, the investigation of the heat exchange of fluids with a constant wall temperature boundary condition, can change an engineer's view in designing heat exchangers including plate heat exchangers. This research deals with numerical investigation of the heat exchange of a nanofluid (with a Non-Newtonian base fluid which has the rheological behavior of Herschel-Bulkley model). As the application of nanofluids especially those with a Non-Newtonian base fluid (such as many lubricating oils whose cooling role in different machinery has been regarded in recent decades) in the matter of heat exchange has not yet completely been recognized, thus, we preferred to use solid nanoparticles in the cooling process of the fluids so that with a growth in the thermal conductivity factor, the heat exchange rate from the fluid to the surface can be increased. In the recent years the use of nanofluids for increasing the heat exchange between fluids and solid surfaces of heat exchangers has become prevalent, so that nowadays in most heat exchange equipment, for a longer durability of components and enhancing the heat exchange rate, different types of nanofluids are utilized due to their high heat exchange factor. In investigating the heat exchange of nanofluids with a Newtonian base fluid (such as water) or Non-Newtonian base fluid (such as different types of industrial oils) two different opinions exist, the assumption of single phase (fluid with solid nanoparticles) and the second is separation of fluid and nanoparticles as liquid and solid phases. Though with the correct determination of equivalent values for density, Viscosity, specific heat capacity and thermal conductivity for the single phase status, we can obtain precise results and there would be no need for a complicated and lengthy two-phase solution. In most papers the single-phase assumption is considered.

1. Introduction
The topic of heat exchange in nanofluids is a very fascinating topic in the world of science and engineering. The term nanofluids introduced by Choi which represents a liquid suspension containing infinitesimal particles (diameter of less than 50nm). Empirical studies have shown that even in very small volume fractions of nanofluids (less than 5%), the heat conductivity of the base fluid can increase as much as10%-50%. The growth of heat conductivity besides the thermal emission of particles and the turbulence induced by their movement causes a considerable enhancement in the heat transfer coefficient which itself leads to the nanofluid creating a more appropriate environment for heat transfer in applications such as cooling of advanced nuclear systems and cylindrical heat pipes .
Kumar and Vinod [1] examined heat transfer using three different non-Newtonian nanofluids comprising of Fe2O3, Al2O3 and CuO nanoparticles in aqueous carboxymethyl cellulose (CMC) base fluid. They found that the CuO/CMC-based nanofluid showed better heat transfer than the other two types of fluid (Fe2O3 and Al2O3). [2] evaluated different concentrations of nanofluids ranging from 0 to 3 wt%. They measured thermophysical properties of nanofluids are used for the estimation of heat transfer characteristics with a Prandtl number range of 228–592. Also, in [3] the turbulent heat transfer and friction factor of nanodiamond-nickel (ND-Ni) hybrid nanofluids flow in a horizontal tube has been investigated experimentally to compare the obtained experimental Nusselt number of hybrid nanofluids with other kind of hybrid nanofluids available.

These studies have details of preparing nanofluids, theoretical and experimental studies of viscosity, conduction of nanofluids and the movement and transfer of nanofluids [4-7]. A criteria study by Hatami et. al., [5] has been on the conduction of nanofluids. In this paper the empirical data of heat transfer gathered from 30 world organizations have been analyzed and a10% compatibility between them is observed. This research concludes that the conduction of nanofluids increases with the density of particles and also relative to their dimensions, which complies with the classic theory of Maxwell which anticipated that the effective heat ratio (k/kf) is dependent upon the solid volume fraction of the nanofluid f and relative heat ratio kp/kf. This functional dependence is valid for Φ << 1 and kp/kf< 10. Another factor of the base fluid which is affected by nanofluids is its viscosity. It has been monitored that the viscosity of nanofluids rises with the rise in the solid volume fraction of the nanofluid. Nonetheless unlike the heat conduction which leads to temperature increase, the viscosity of nanofluids barely changes in temperature fluctuations of 25-89 degrees Celsius. Numerous ideas have been expressed to depict the identity of additional heat transfer in nanofluids. Pak and Cho describe the growth in the heat transfer of nanofluids as a result of emission of particles. Khan and Lee suggested that the increase in the heat transfer of nanofluids is due to the turbulence induced by the movement of nanoparticles. After experiments on water and glycerin based nanofluids, Ahouja concluded that the increase in heat transfer is because of the spinning of nanoparticles. Though after meticulous evaluations, Bounjiormo deduces that the high heat transfer cannot be the result of heat emission, rise of turbulence or the spinning particles. He explains that the analytical model for the movements in nanofluids must be considered in Browny and Thermophoresis distribution and that the excessive heat transfer is because of the considerable decline in the fluids viscosity due to the large temperature changes in the boundary layers .
Research in the field of convective heat transfer has been permanently advancing in the past ten years. Nonetheless the analytical studies on convective heat transfer in nanofluids are much less, compared to studies in forced heat transfer of nanofluids [6]. Khanafar et. al., [7] analyzed the natural two-dimensional convection of nanofluids in a crate and understood that for the Grashof number given, the heat transfer rate increases with the increasing solid volume fraction of nano-particles. In this research, the effects of Reynolds and Rayleigh Numbers on the Non-Newtonion Nanofluid (Al2O3/Propellant Dough) heat transfer investigated.

2. Research Method

In the recent years the flow and heat exchange behavior of Non-Newtonian fluids specifically the fluids of Herschel-Bulkley model have been under scrutiny because of their huge application in different industries such as, food industry, polymer melts, pharmaceutical industry, biotechnology industries and manufacturing of pulp, natural and artificial rubber solutions. The Herschel-Bulkley fluids are the group of Non-Newtonian fluids which have a non-linear relation between tension and strain.

$$\tau = \mu\gamma^n + \tau_y \qquad \tau \geq \tau_y (1)$$

$$\gamma = 0 \qquad \tau < \tau_y (2)$$

In the equations (1) and (2), *n* (the power-law characteristic) and τ_y (the yielding tension) are the rheologic properties of the fluid. In this research the Propellant Dough(35/45) as a Herschel-Bulkley fluid with the thermophysical characteristics mentioned in the table below was selected as the base fluid for the study.

The Richardson and Grashov numbers are calculated from the equations below :

$$Ri = \frac{Gr}{Re^2} (3)$$

$$\text{Gr} = \frac{g(\beta)_f(T_w - T_\infty)L^3}{\vartheta^2} (4)$$

From the equations above we can conclude that for this problem, $\frac{Gr}{Re^2} << 1$ and therefore the forced movements can be ignored and regarding the Reynolds number assumed (Re $\approx 10^{-3}$) and the type of fluid as a base fluid (fluid with high viscosity) and the creeping flow theory which is prevailing over this case, we can consider the movements as natural movements compare the results with the results of research done in field of natural movement.

Table 1: The Thermophysical characteristics of Propellant Dough(35/45) and Nanoparticles [8]

Parameters / Material	Consistency (Nsm^{-2})	Thermal conductivity ($Wm^{-1}K^{-1}$)	Temperature-shift fxtorfor the viscosity (K^{-1})	Power-Law Index (n)
Propellant Dough(35/45)	17900	0.34	0.0437	0.440
Propellant Dough(35/45)	4330	0.34	0.0437	0.466
Propellant Dough(35/45)	1270	0.34	0.0437	0.533
Al_2O_3	-	40	$0.85*10^{-5}$	-
Parameters / Material	**Temperature (K)**	**Density (kgm-3)**	**Specific heat capacity (Jkg-1 K-1)**	**Yeild stress (kPa)**
Propellant Dough(35/45)	293.15	840	1000	39.0
Propellant Dough(35/45)	308.15	840	1000	47.5
Propellant Dough(35/45)	323.15	840	1000	50.0
Al_2O_3	-	3970	765	-

3. The Equations Dominating the Problem

For describing the flow of a slimy and incompressible fluid, the laws of mass conservation, momentum and energy conservation are used. For the flow of a nanofluid, the equation of mass conservation (5), momentum conservation (6) and energy conservation (7) are depicted below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (5)$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\left(\vartheta_{nf}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+(\beta)_{nf}g(T-T_\infty) \qquad (6)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (7)$$

All the equations are written for a uniform status.

Density, diffusion, heat capacity and thermal expansion coefficient of the nanofluid are in equations (8) to (11) respectively:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p \qquad (8)$$

$$\alpha_{nf}=k_{nf}/(\rho c_p)_{nf} \qquad (9)$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_p \qquad (10)$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_p \qquad (11)$$

For consistency of the nanofluid, three equations have been expressed in which the first equation (12) is the Brinkman formula and the other two (13) and (14) are empirical formulas.

$$\mu_{nf}=\mu_f/(1-\phi)^{\frac{2}{5}} \qquad (12)$$

$$\mu_{nf} = \mu_f(1 + 39.11\phi + 533.9\phi^2) \quad (13)$$

$$\mu_{nf} = \mu_f(1 + 7.3\phi + 123\phi^2) \quad (14)$$

For the thermal conductivity of the nanofluid, 2 equations are stated. The first one (15) is the Maxwell formula and the next one (16) is an empirical formula.

$$k_{nf} = k_f\left[\frac{(k_p+2k_f)-2\phi(k_f-k_p)}{(k_p+2k_f)+\phi(k_f-k_p)}\right] \quad (15)$$

$$k_{nf} = k_f(1 + 7.47\phi) \quad (16)$$

The boundary conditions for this case are :

$$y = 0 \ , \ u = v = 0 \ , \ T = T_w$$

$$y \to \infty \ , \ u \to U_\infty \ , \ T \to T_\infty \quad (17)$$

Defining the dimensionless variables :

$$X = \frac{x}{L}, Y = \frac{y}{L}Re^{1/2}, U = \frac{u}{U_\infty}, V = \frac{v}{U_\infty}Re^{1/2}, \eta = \frac{y}{x}{Ra_x}^{1/4}, \theta = \frac{T - T_\infty}{\Delta T}$$

$$P = \frac{\bar{p}L^2}{\rho_{nf}\alpha_f^2}, Ra = \frac{g\beta_f L^3\Delta T}{\nu_f\alpha_f}, Re = \frac{U_\infty L}{\vartheta}, Gr = \frac{g\beta_f L^3\Delta T}{\vartheta^2}, Pr = \frac{\nu_f}{\alpha_f}, Br = \frac{\mu_f u^2}{k_f(T_w - T_\infty)} \quad (18)$$

Mass conservation equations, Momentum conservation and Energy conservation are rewritten dimension less in the form below :

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (19)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \left(\frac{\partial^2 U}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta \quad (20)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (21)$$

The boundary conditions are rewritten as dimensionless :

$$Y = 0 \ , \ U = V = 0 \ , \ \theta = 1$$

$$Y \to \infty \ , \ U \to 1 \ , \ \theta \to 0 \quad (22)$$

The local Nusselt number on the surface of the heat source is driven from the equation below :

$$Nu_s = \frac{hL}{k_f} \quad (23)$$

The local thermal exchange coefficient (h) is defined as below :

$$h = \frac{q''}{T_w - T_\infty} \quad (24)$$

The average thermal exchange coefficient ($\bar{h}$) is defined as below :

$$\bar{h} = \frac{1}{L}\int_0^L x \, dx \quad (25)$$

With making the parameters dimensionless and applying the equations (23),(24) can be rewritten as seen below:

$$Nu_w(X) = \frac{1}{\theta_w(X)} \quad (26)$$

The surface friction coefficient and the shear stress on the wall can be calculated from the equations below :

$$C_f = \frac{\tau_w}{0.5\rho v^2} \quad (27)$$

$$\tau_w = \mu \frac{\partial u}{\partial y} \quad (28)$$

The local and average Nusselt according to Reynolds and Rayleigh number in a laminar flow with a constant wall temperature boundary condition can be calculated from the equations below :

$Nu = 0.503(Ra)^{1/4}$(for high Prandtl numbers) (29)

$Nu_x = 0.332(Re_x)^{0.5} (Pr)^{1/3}(Re<5*10^5 \;, \; Pr\geq 0.6)$ (30)

$Nu = 0.664(Re_x)^{0.5} (Pr)^{1/3}(Re<5*10^5 \;, \; Pr\geq 0.6)$(31)

4. Results

4.1. The effect of Reynolds on Nusselt number and surface friction coefficient

The value of Nusselt number and friction coefficient on the vertical plane is investigated. According to the following diagrams it is obvious that when Reynolds is increased, Nusselt number and so thermal transfer and surface friction coefficient is reduced.

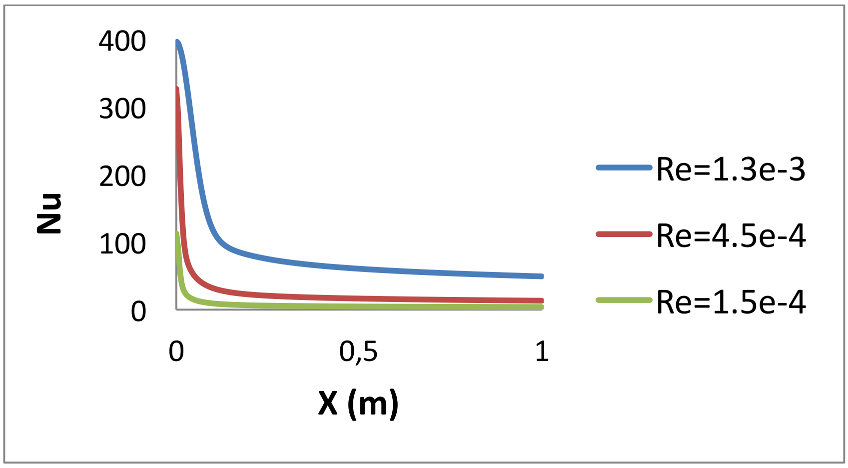

Figure 1: Nusselt number profilealong the vertical wall for the volume fraction of 7% of nanoparticles (Nanofluids Al_2O_3/Propellant Dough(35/45)) – the fluid temperature of 20°C and temperature difference of fluid and wall 24°C and n=0.440

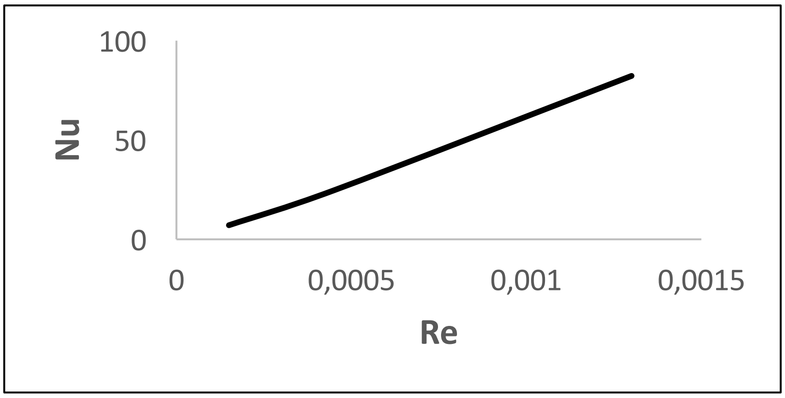

Figure 2: The Average Nusselt Number Profile in the length of the vertical wall according to Reynolds number for the volume fraction of 7% of nanoparticles (Nanofluids Al_2O_3/Propellant Dough(35/45)) – the fluid temperature of 20°C and temperature difference of fluid and wall 24°C and n=0.440

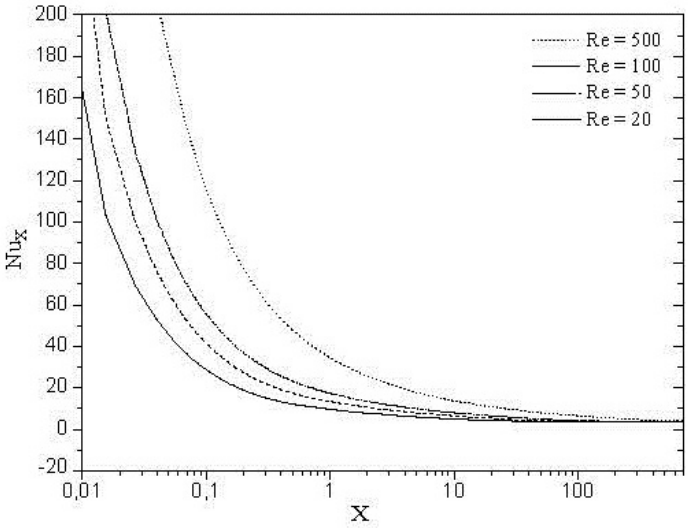

Figure 3: Nusselt Number Profile in different Reynolds numbers for a Herschel-Bulkley fluid [9]

The above chart confirms the results obtained in this article.

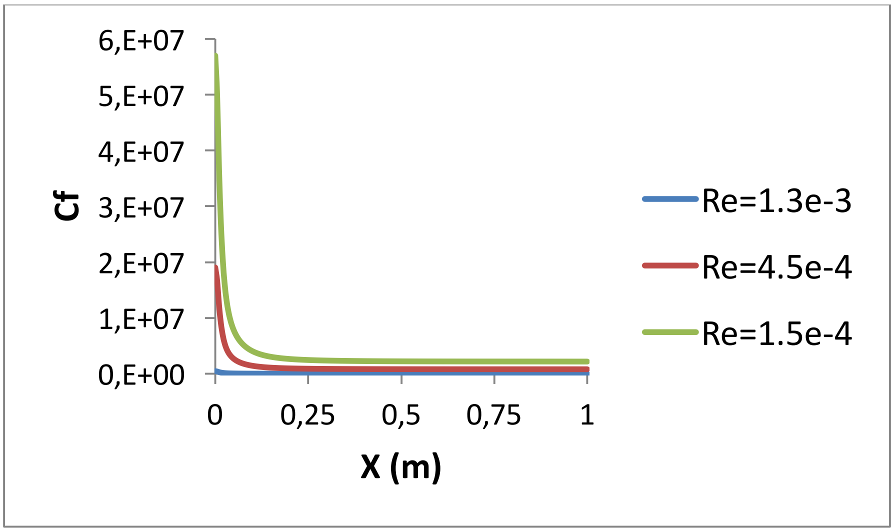

Figure 4: Surface Friction Coefficient profilealong the vertical wall for the volume fraction of 7% of nanoparticles (Nanofluids Al_2O_3/Propellant Dough(35/45)) – the fluid temperature of 20°C and temperature difference of fluid and wall 24°C and n=0.440

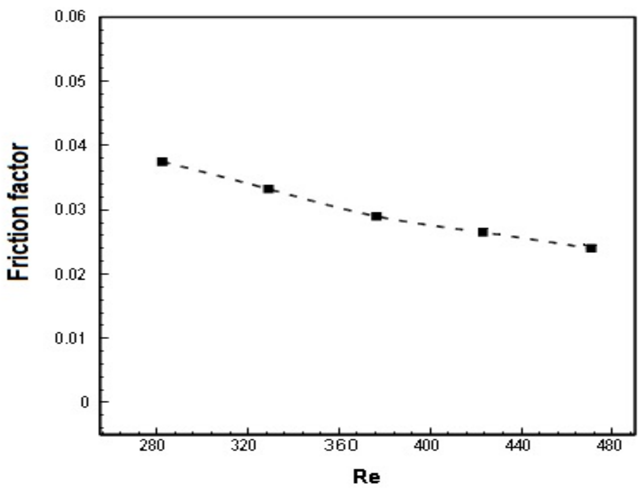

Figure 5: Surface Friction Coefficient Profile according to Reynolds numbers for a nanofluid [10]

The above chart confirms the results obtained in this article.
According to equation (30), Nusselt number has a direct relation with Reynolds number and so in the above diagram with the increase of Reynolds number, Nusselt number is also increased. As well, with the increase of Reynolds number velocity increases, according to equation (27) velocity has a reverse impact on surface friction coefficient and so in the above diagram with the increase of Reynolds number surface friction coefficient is decreased.

4.2. The effect of Rayleigh number changes on Nusselt number

As is shown in the following diagram the increase of Rayleigh number leads to the increase of Nusselt number.

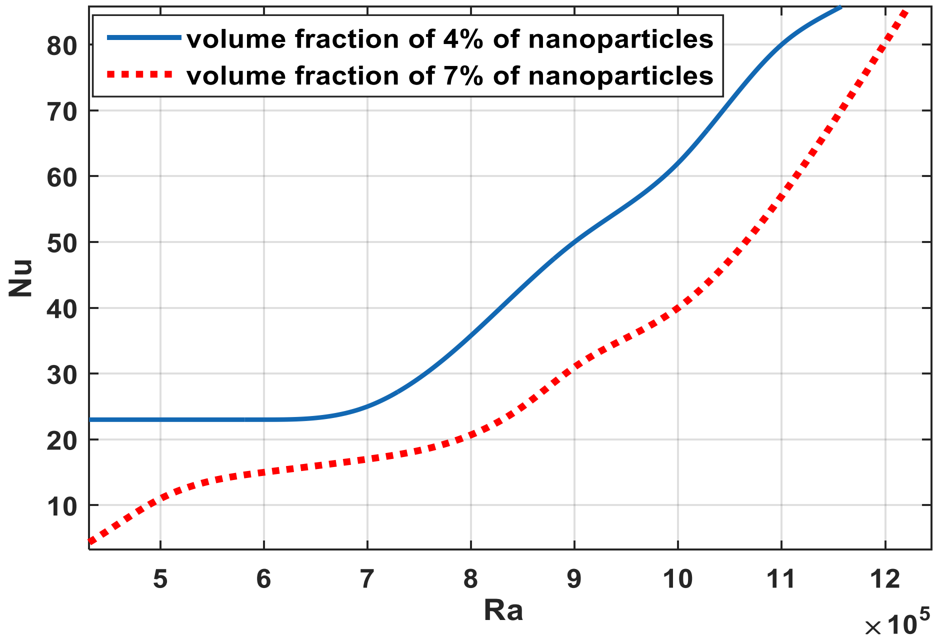

Figure 6: Nusselt number profilealong the vertical wall according to Rayleigh number for the volume fraction of 4% and 7% of nanoparticles (Nanofluids Al_2O_3/Propellant Dough(35/45)) – n=0.440

In the boundary condition of constant temperature on the wall, the increase of Rayleigh number makes temperature difference of fluid and wall to be increased and so thermal transfer and average Nusselt number increase.
According to the following tables for the fluid used in this research the Nusselt numbers obtained from empirical equation (29) for boundary condition of constant heat flux on the vertical wall are compatible with results of numerical solutions in this research with minor error.

Table 2: Comparison of Nusselt numbers obtained from empirical equation and numerical solution in the research – the boundary condition of constant heat flux on the vertical wall, the temperature of fluid is 20°C, temperature difference between fluid and wall is 24°C and Reynolds number is $1.3*10^{-3}$

	Nusselt number obtained from equation (29)	Nusselt number obtained from numerical solution in this research	Relative error
the volume fraction of 4% of nanoparticles	54.33	54.75	%0.77
the volume fraction of 7% of nanoparticles	40.71	40.49	%0.005

Discussion and conclusions

In this study, combined convection on a vertical plate with the boundary condition of constant wall heat flux on the wall with non-Newtonian nanofluid of Al_2O_3/Propellant Dough(35/45) is investigated numerically. The flow is assumed to be laminar, stable, incompressible, viscose and two-dimensional. The effects of Reynolds and Rayleigh numbers in the volume fraction of 7% on local and average Nusselt number and surface friction coefficient is studied. The important results obtained from this study are as following:

1- When Reynolds number is increased, Nusselt number and so thermal transfer is increased.
2- Increase of Reynolds number leads to the decrease of surface friction coefficient.
3- Increase of Rayleigh number has led to the increase of Nusselt number.

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