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New solutions for estimation of critical depth in trapezoidal cross section channel

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Abstract
Critical depth is an important parameter in design of open channels and analysis of gradually varied flow. In trapezoidal channels, the governing equations are nonlinear for computing critical depth. Therefore, implicit solution of the equations involves numerical methods. Any methods that researchers have suggested for solving these equations are so complex. In this paper, two simple and useful semi-analytical solutions were introduced using Data Fit software that has minimum errors in calculating the critical depth in trapezoidal channels. In first solutions, equation (20) was obtained based on the equation of degree 6 for trapezoidal channels (Equation 4). In second solutions, critical depth equation in trapezoidal channel was developed as the equation (22) using critical depth equation in triangular channel. Results show these equations have acceptable accuracy, which relative error of equations (20) and (22) are 1.22% and 1.34% respectively.

Key words: Hydraulic parameter, Numerical solution, Relative error, trapezoidal channel, Triangular channel.

1. Introduction

Critical and normal depths values are used in designing the open channels and analysis the gradually varied flow profiles [1]. Also, flow discharge in open channels is determined using the critical depth of control section. Critical depth is calculated using trial and error methods except for triangular and rectangular sections [2, 4-6]. Generally, critical depth in rectangular channel is calculated using the following equation:

$$y_c = \left(\frac{\alpha q^2}{g}\right)^{1/3} \tag{1}$$

Where: y_c is critical depth (m), g is gravity acceleration (m/s^2), q is discharge per unit width (m^2/s) and α is energy correction factor. Also, critical depth in triangular channels is calculated with the following equations:

$$y_c = \left(\frac{2\alpha Q^2}{gm^2}\right)^{1/5} \tag{2}$$

Where: m is side slope (H:V). Critical depth could not be directly calculated in trapezoidal channel. The main equation for computing critical depth in a simple trapezoidal section (Figure 1) is as [2, 4-6]:

$$\frac{\alpha Q^2 T_c}{g A_c^3} = 1 \tag{3}$$

Where: T_c is width of the channel at the water surface in critical condition (m) and A_c is area of flow cross section in critical condition (m^2). By substituting T_c and A_c into equation (3),it could be written as [2,4 and 5]:

$$(y_c(b + my_c))^3 - \frac{\alpha Q^2}{g}(b + 2my_c) = 0 \tag{4}$$

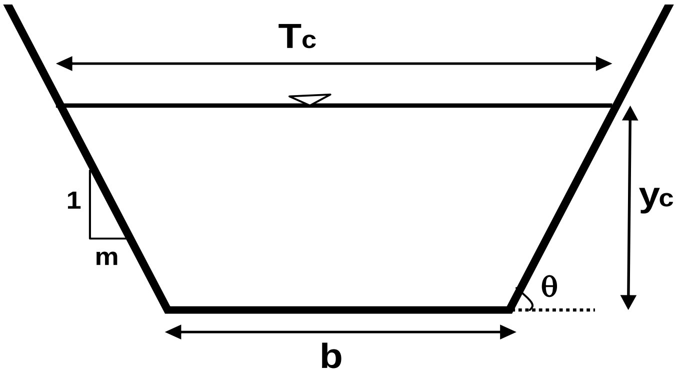

Figure 1: Geometric and hydraulic parameters of trapezoidal channel in critical condition

For calculating the critical depth in trapezoidal channel, equation of degree 6 (Equation 4) must be solved. But any analytical solution method has not been proposed yet. Researchers have suggested several methods for solving this problem. Wang [10] suggested a non-dimensional equation for calculating the critical depth of trapezoidal channels as:

$$x(1+x) = k(1+2x)^{1/3} \tag{5}$$

$$x = \frac{my_k}{b}, \quad k = \frac{my'_k}{b}, \quad y'_k = \sqrt[3]{\frac{\alpha q^2}{g}} \tag{6}$$

Where: y_k is critical depth of trapezoidal channels (m) and y'_k is critical depth of rectangular channel that the channel bottom width is equal to trapezoidal channel(m). For solving equation (5), Wang [10] developed the following equation:

$$y_k = \frac{b}{2m}\left(\sqrt{1+4k\left\{1+4k\left[1+4k(1+4k)^{1/5}\right]^{1/6}\right\}^{1/6}} - 1\right) \tag{7}$$

Equation (4) is often written as [8]:

$$t_c^6 - \varepsilon_c t_c - 1 = 0 \tag{8}$$

Where: $t_c = (2\eta_c + 1)^{1/3}$, $\eta_c = \frac{my_c}{b}$ *and* $\varepsilon_c = 4\times(\frac{\alpha m^3 Q^2}{gb^5})^{1/3}$. Wang et al. [11] used equation (8) and developed equation of critical depth in trapezoidal channels as:

$$\eta_c = \frac{my_c}{b} = \frac{\sqrt{1+\varepsilon_c(1+\varepsilon_c)^{0.2}} - 1}{2} \tag{9}$$

Vatankhah and Easa [9] proposed equation (8) on the other hand as:

$$y_c = 0.25\frac{b}{m}\varepsilon_c(1+0.2722\varepsilon_c^{1.041})^{-0.339}, \quad 0 \le \varepsilon_c \le 25 \tag{10}$$

Vatankhah [8] solved equation (8) by Newton-Raphson method and reported the following equation:

$$\eta_c = -\frac{1}{2} + \frac{1}{2}\left(\frac{5t_{c0}^{0.864} + 1}{6t_{c0}^{0.720} - \varepsilon_c}\right)^3 \tag{11}$$

Where: $t_{c0} = 1 + 1.161\varepsilon_c(1+0.666\varepsilon_c^{1.041})^{0.374}$.

Also, Swamee and Rathie [7] have reported exact analytical equations for critical depth in trapezoidal cross sections. Most of the equations suggested for calculating critical depth of trapezoidal channels have complex forms and are used in special condition. In this research, two different solutions are introduced and evaluated in calculating critical depth of trapezoidal channels.

2. Materials and methods

To obtain the equations of critical depth in trapezoidal channel, 17100 critical depths were generated in Microsoft Excel. In Table (1), variation of geometric and hydraulic parameters was shown. Also, in order to test the equations, 900 critical depths were approximately generated in trapezoidal channel. It was attempted to use actual data in depth generation process, so that it was observed $1 \le \frac{b}{y} \le 6$ condition.

Table 1: Variation of geometric and hydraulic parameters

Parameters	Min	Max
$b(m)$	0.6	6
m	1	3
α	1	1.09
$y_c(m)$	0.15	4
$Q(m^3/s)$	0.1193	349.3

In this research, two solutions are suggested for calculating the critical depth in trapezoidal channels. First solution is in conjunction with the equation of degree 6 for trapezoidal channels (Equation 4). Second solution is a new method that is obtained using the critical depth equation in triangular channels.

2.1 First solution

In the equation (4), with assuming $\lambda = \frac{my_c}{b}$ *and* $y_c = \frac{b\lambda}{m}$, equation (4) could be rearranged as:

$$\frac{\lambda^3(1+\lambda)^3}{1+2\lambda} = \frac{\alpha Q^2 m^3}{b^5 g} \tag{12}$$

With assuming $K = \frac{\alpha Q^2 m^3}{gb^5}$, equation (12) could be written as:

$$\lambda^6 + 3\lambda^5 + 3\lambda^4 + \lambda^3 - 2\lambda K - K = 0 \;\Rightarrow\; \lambda = f_1(K) \tag{13}$$

In equation (13), K and λ are non-dimensional parameters. Value of K is well-known and value of λ could be calculated. However, equation (13) has no any analytical solution yet. By using the generated critical depths and equation (13), a new equation was suggested to calculate critical depth in trapezoidal channels.

2.2 Second solution

In this method, critical depth in trapezoidal channels is calculated using critical depth equation in triangular channels. With attention to Figure (2) and assumption critical condition in OAB triangle:

$$b = 2mh \quad , \quad h = \frac{b}{2m} \tag{14}$$

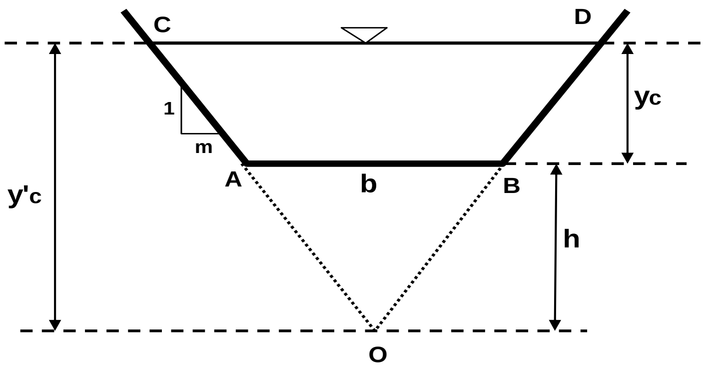

Figure 2: Geometric and hydraulic parameters in critical condition

And based on the equation (2):

$$h=\left(\frac{2\alpha Q'^2}{gm^2}\right)^{1/5}\Rightarrow Q'=\frac{\sqrt{\frac{g}{\alpha}}\,b^{2.5}}{8m^{1.5}} \tag{15}$$

Where: Q' is discharge in OAB triangle with assumption critical condition $\left(m^3/s\right)$. With these assumptions, flow discharge in OCD triangle is equal to:

$$Q''=Q+Q' \tag{16}$$

Where: Q'' is flow discharge in OCD triangle $\left(m^3/s\right)$ and Q is flow discharge in ABCD trapezoid $\left(m^3/s\right)$.

It is necessary to mention that flow conditions are critical in the cross section. Based on the equation (2), critical depth in OCD triangle is:

$$y'_c=\left(\frac{2\alpha Q''^2}{gm^2}\right)^{1/5} \tag{17}$$

And Critical depth in ABCD trapezoid can be fitted as:

$$y''_c=y'_c-h=y'_c-\frac{b}{2m} \tag{18}$$

Using dimensional analysis of the critical depth in trapezoidal channel, the following function is obtained:

$$\frac{y_c}{y''_c}=f_2\left(\frac{Q''}{Q}\right) \tag{19}$$

Finally, critical depth in trapezoidal channel can be calculated using the equation (19). In the following, results of the methods are reported and compared together.

3. Results and discussion

3.1 First solution results

In this method, by using equation (13) and 17100 generated critical depths; the flowing equation was obtained in *Data Fit* software:

$$\lambda=-1.55\,K^{0.06}+1.68\,K^{0.18}+0.644 \tag{20}$$

Where: $\lambda=\frac{my_c}{b}$ *and* $K=\frac{\alpha Q^2 m^3}{gb^5}$.

Figure (3) shows calculated non-dimensional parameters of critical depth in trapezoidal channel (λ_E) using equation (20) versus real non-dimensional parameter of critical depth (λ_P). As shown in this figure, equation (20) can calculate the non-dimensional parameter of critical depth (λ_E) well. Relative percentage error (RPE) was calculated by:

$$\mathrm{RPE\%}=\frac{|y_{cP}-y_{cE}|}{y_{cP}}\times 100 \tag{21}$$

In equation (21), the subscript “P” describes real parameter and the subscript “E” describes calculated parameter by using equation (20). Maximum relative percentage error of equation (20) was obtained 0.97%.

3.2 Second solution results

Most of the known methods for computing the critical depth of trapezoidal cross section are based on equation (13). But second solution is a new and different method. This solution suggested in this research, is based on critical depth equation in triangular channels. Using 17100 generated depths and equation (19), the following equation was fitted in *Data Fit* software:

$$\frac{y_c}{y''_c}=\frac{2.57\times\left(\frac{Q''}{Q}\right)}{1.53+\left(\frac{Q''}{Q}\right)} \tag{22}$$

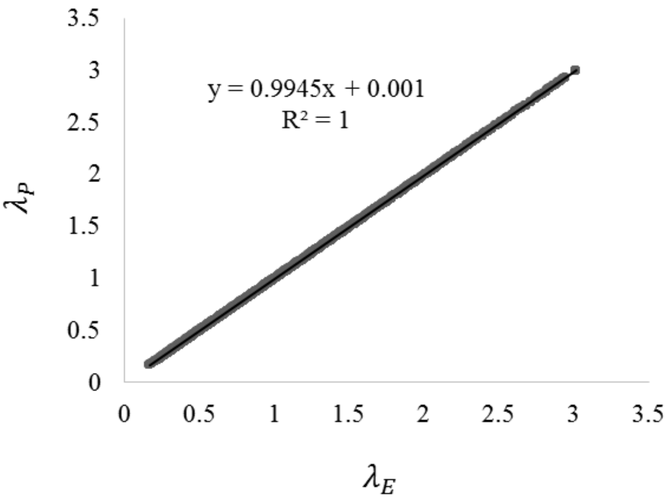

Figure 3: Calculated non-dimensional parameter of critical depth (λ_E) -Equation (20) - versus real non-dimensional parameter of critical depth (λ_P)

Error of equation (22) to calculate critical depth is negligible (Figure 4). Otherwise, maximum relative percentage error of equation (22) was 0.95%. Figure 4 shows that equation (22) can calculate critical depth of trapezoidal channel well. Flow discharge value is the most important parameter in channel designing and the parameter is the main one in the form of equation. In equation (22), critical depth of trapezoidal channel could be simply calculated with high accuracy without trial and error. The proposed equation has simple form, easy calculation and wide application range compared with the existing equations.

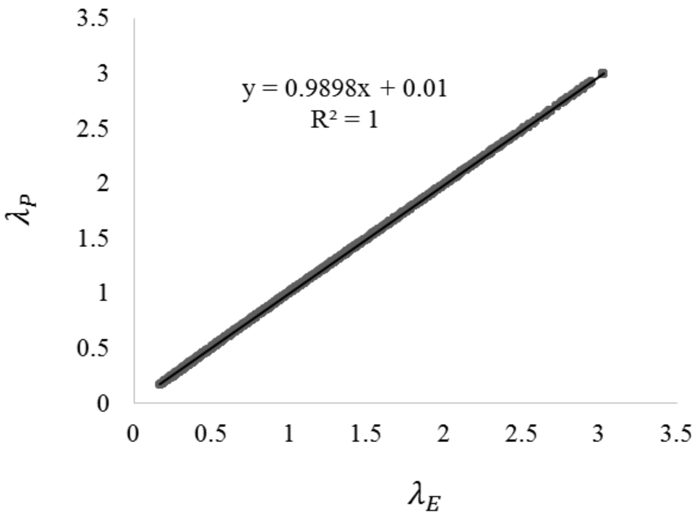

Figure 4: Calculated non-dimensional parameter of critical depth (λ_E) -Equation (22) - versus real non-dimensional parameters critical depth (λ_P)

In the test stage, calculated non-dimensional parameter of critical depth using equations (20) and (22) were compared with real data. Results show these equations have acceptable performance, which for 900 generated depths, maximum relative percentage error of equations (20) and (22) are 1.22% and 1.41% respectively (Figures 5 and 6). Figures (7), (8) and (9) depict calculated non-dimensional parameter of critical depth (λ_E) - Wang's equation, Vatankhah and Easa's equation and Vatankhah's equation- versus real non-dimensional parameters of critical depth (λ_P). As shown in Figures (7), (8) and (9), results of the equations have good agreement with the real data and the equations are accurate solution. But according to Vatankhah and Easa, Wang's equation is complicated solution [9]. Also, maximum relative error of Wang's equation,Vatankhah and Easa's equation and Vatankhah's equation are 0.0154%, 1.63% and 5.5E-06% respectively.

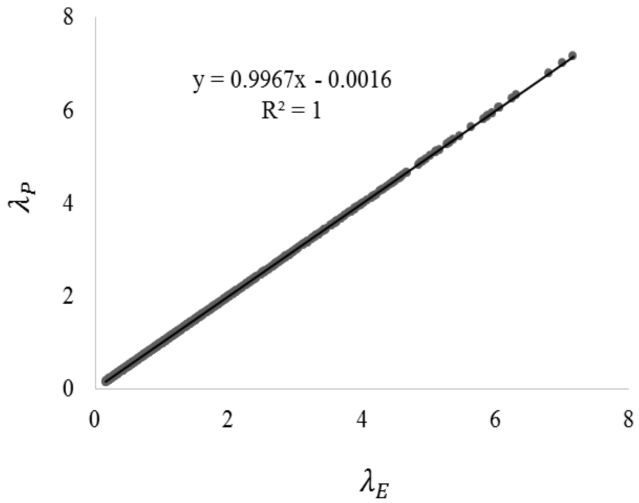

Figure 5: Calculated non-dimensional parameter of critical depth (λ_E) -Equation (20) - versus real non-dimensional parameter of critical depth (λ_P) in test stage

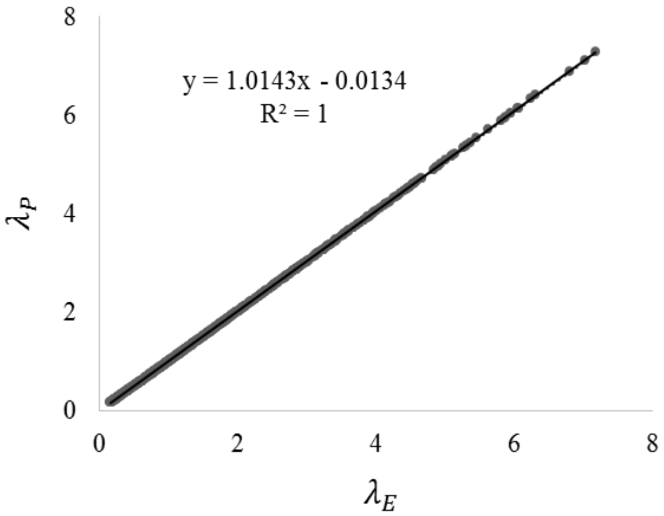

Figure 6: Calculated non-dimensional parameter of critical depth (λ_E) -Equation (22)- versus real non-dimensional parameters critical depth (λ_P) in test stage

Configuration of Equation (20) is similar to the pervious equations with two non-dimensional parameter including $\lambda = \eta_c$ and $K = (\frac{\varepsilon_c}{4})^{1/3}$ .So that Equation (20) predicts the critical depth as well as the pervious equations. In the polynomial form, the more complex the curvature of the data, the higher the polynomial order required to fit it. There are no data restrictions associated with this equation form. But for extrapolating beyond the range of the data, polynomial models can change direction suddenly beyond the range of the data. Otherwise, it offers up to a six to order equation. The higher order equations have more inflection points[3]. Equation (22) has a different ground work that calculates critical depths with acceptable accuracy. This equation is known as hyperbola formula that computational steps are lower and simpler than the other equations. Otherwise, the proposed equation is very user-friendly and applicable, but the other researchers' equations have complex and difficult forms. It is concluded that non-dimensional forms of the governing equation are very powerful tools for developing general explicit regression-based equations [8].

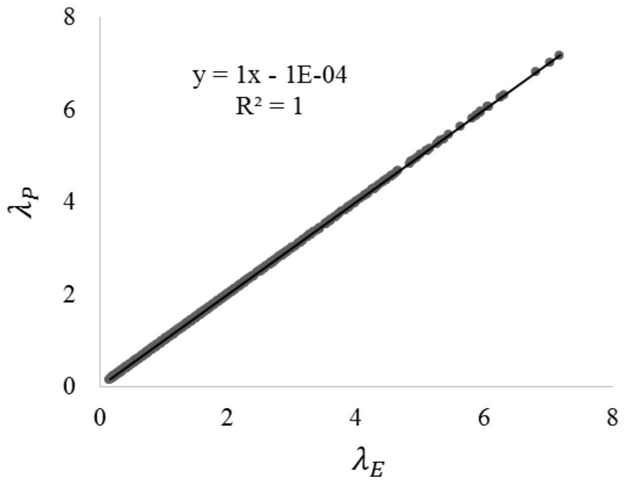

Figure 7: Calculated non-dimensional parameter of critical depth (λ_E) -Wang's equation- versus real non-dimensional parameters critical depth (λ_P) in test stage

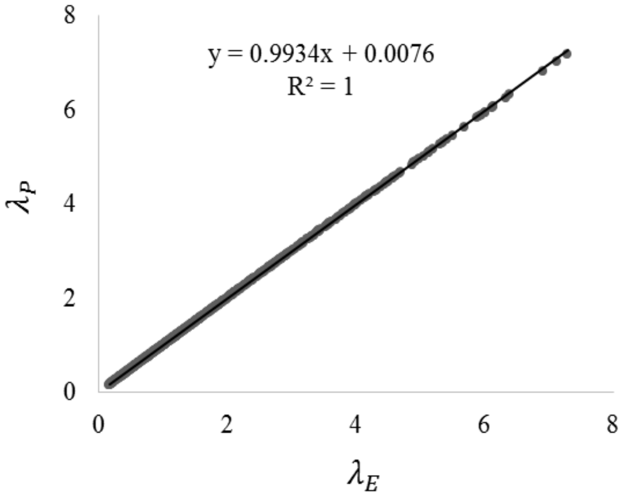

Figure 8: Calculated non-dimensional parameter of critical depth (λ_E) -Vatankhah and Easa's equation- versus real non-dimensional parameters critical depth (λ_P) in test stage

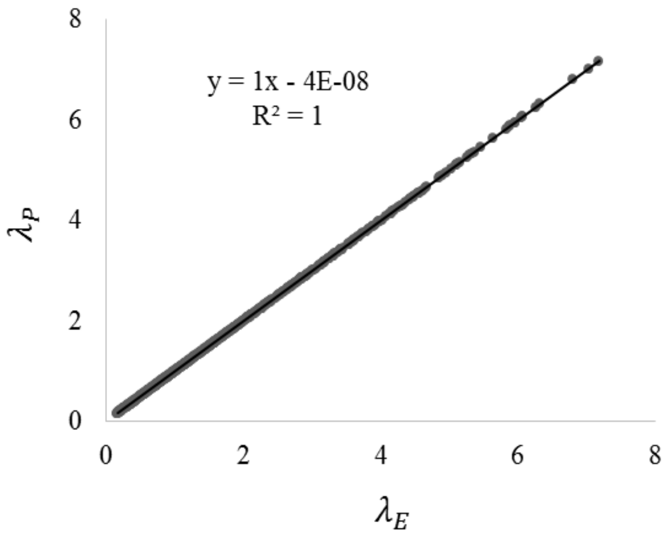

Figure 9: Calculated non-dimensional parameter of critical depth (λ_E) -Vatankhah's equation- versus real non-dimensional parameters critical depth (λ_P) in test stage

Conclusion

1. In this research, two solutions were used to calculate critical depth in the trapezoidal channels.
2. First solution was based on the analytical method and using equation (13), equation (20) was fitted in *Data Fit* software.
3. Second solution was a new and different method that was based on the critical depth in triangular channel. In this method, equation (22) was fitted in *Data Fit* software.
4. Results showedthat the maximum relative percentage error of equations (20) and (22) are 1.22% and 1.41% respectively (Figures 5 and 6).
5. Calculaed critical depths using Wang's equation, Vatankhah and Easa's equation and Vatankhah's equation showed that the proposed equations (20) and (22) had acceptable accuracy as well as the other equations.
6. The proposed equations were very simple and easier to solve (user-friendly) that could calculate critical depth of trapezoidal channel more accurately.

References

1. Achour B., Khattaoui M., *Open Civil Eng. J.*, 2 (2008) 9.
2. Chow V. T., Open channels hydraulics, *McGraw-Hill*, New York. (1973).
3. Draper N.R., Smith H., Applied regression analysis, *Wiley-Interscience*, New York. (1998).
4. Henderson F. M., Open channel flow, *Mac Millan*, New York. (1966).
5. French R. H., Open cannel hydraulics, *McGraw-Hill*, New York. (1986).
6. Swamee P. K., *J. Irrig. Drain. Eng.* 119 (1993) 400.
7. Swamee P. K., Rathie P. N., *J. Irrig. Drain. Eng.*, 131 (2005) 474.
8. Vatankhah A. R., *Ain Shams Eng. J.* 4 (2013) 17.
9. Vatankhah A. R., Easa S., *Flow Meas. Instrum.* 22 (2011) 9.
10. Wang Z. Z., *J. Hydraul. Eng.,* 124 (1998) 90.
11. Wang Z. Z., Yuan S., Wu C.L., *J. Hydraul. Eng.*, 43 (1999) 7, (In Chinese).

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