



## Computational modelling of tidal effect on wastewater dispersion in coastal bays, case of Tangier's bay (Morocco)

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### Abstract

This study is focused on numerical simulation of the process of coastal pollution by wastewater in the bay of Tangier. The numerical model is based on shallow water equation used for the hydrodynamic and equation of advection-diffusion for describes the wastewater assimilated like passive pollutant. The obtained model is resolved using an unstructured triangular finite volume method. The found results show that: (i) the tidal flows have a direct effects on wastewater dispersion that can used to control its evacuation out of bay (ii) the numerical model used can modelling domains that have more complicated geometry and bathymetry, this in a large time scale.

*Key words:* Pollutant dispersion, Numerical modeling, Tanger's bay, Shallow water equation.

### 1. Introduction

The environment of the Tangier's Bay is subjected to the nearby impact of human activity, due to urban development: industrial, agricultural, fisheries and port activity, including that bound to the newly Tangier Mediterranean Port. These activities release toxic effluent are causing ecological damage to the bay.

In the literature, there are several examples of modeling for pollutant transport in the bay of Tangier. Among others, Chaabelasri et al. [1,2] proposed an adaptive finite volumes method for modelling the pollutant transport in coast Tangier's bay; for lagoons area. Chaabelasri et al. [3], Benkhaldoun et al. [4] and Brenon et al. [5] proposed an integrated numerical model for tracers transport and hydrodynamics description.

In the current work we will model the tidal effect on wastewater dispersion in coast Tangier's bay. The used mathematical model is based on the two-dimensional shallow water equations, coupled with the advection-diffusion equation for modeling both current and pollutant transport, and not forgetting the equation of astronomical tide for describing the coming water flow. The finite volume method used to solve the governing equations is based on an unstructured triangular mesh with the Roe-MUSCL scheme used to compute the convective fluxes [6], and a Vazquez-Cendon scheme for numerical description of the source terms [1].

This paper is organized as follows: Section 2 describes the study area, section 3 briefly outlines the governing equations and the basis numerical model, section 4 presents the founded numerical results and their discussions, and finally, the conclusions are summarized in section 5.

### 2. Study area

Tangier's bay (Figure 1) is a semi-enclosed shallow basin located in the north-western extremity of Morocco, on the southern border of the Gibraltar Strait, between parallels 35°46' and 35°48' North and meridians 5°45' and 5°49' West. It has a dense river network in the form of low rivers flowing through the city from south to north. The intensity of their flow is especially remarkable in their upstream course while downstream they are almost perennial, because of drainage of wastewater from the city. Moreover, water volume of these rivers can make a significant volumetric flow contribution during rainy seasons, causing severe flooding, affecting especially neighborhoods lying in low areas of the city because of the impermeability of the soil and the steep slopes of surrounding hills.

### 3. Model description

#### 3.1. Mathematical models:

In the Tangier's bay the domain is shallow, moreover, the mathematical description of the flow hydrodynamics is provided by the two-dimensional (2D) shallow water equations. These are coupled with advection-diffusion

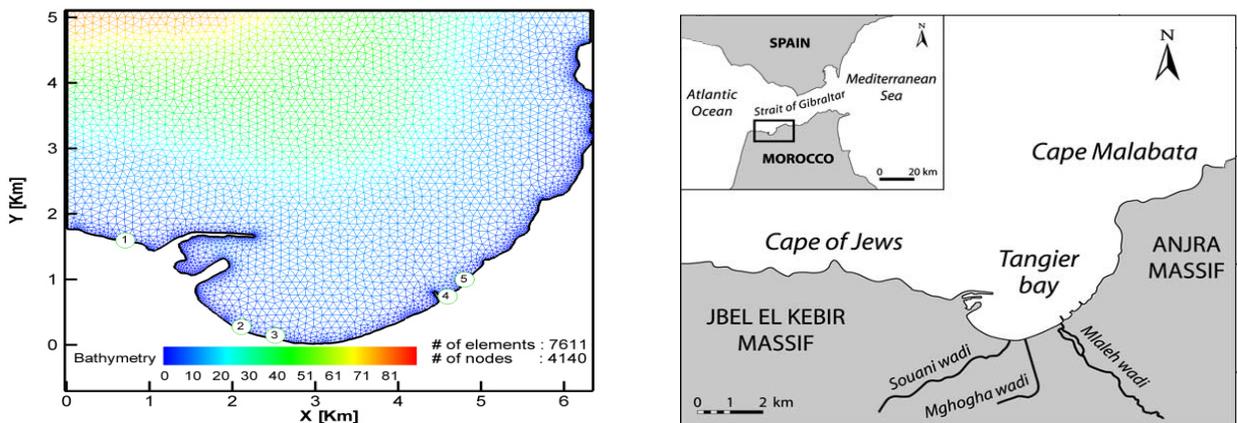
equation to represent the transport of waterborne contaminants. The resulting equation system is written in tensor notation as [7]:

$$\frac{\partial h}{\partial t} + \frac{\partial(hu_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial(hu_i)}{\partial t} + \frac{\partial(hu_i u_j)}{\partial x_j} = -g \frac{\partial}{\partial x_i} \left( \frac{h^2}{2} \right) - gh \frac{\partial Z_b}{\partial x_i} - \frac{\tau_{bi}}{\rho} + \frac{\tau_{wi}}{\rho} \quad (2)$$

$$\frac{\partial(hC)}{\partial t} + \frac{\partial(u_i hC)}{\partial x_i} = \frac{\partial}{\partial x_i} (D_i \frac{\partial(hC)}{\partial x_i}) + S_c \quad (3)$$

where  $i$  and  $j$  are indices and the Einstein summation convention is used,  $x_i$  is the Cartesian coordinate;  $h$  is water depth;  $t$  is time;  $u_i$  is the depth-averaged velocity component in the  $i$ -th direction;  $Z_b$  is the bed elevation above a fixed horizontal datum;  $g = 9.81 m/s^2$  is gravitational acceleration;  $\rho$  is water density,  $C$  is depth-averaged concentration,  $D_i$  is the dispersion coefficient in the  $i$ -th direction,  $S_c$  is the depth-averaged source term;  $\tau_{bi}$  is bed shear stress in the  $i$ -th direction defined by  $\tau_{bi} = \rho C_b u_i \sqrt{u_j u_j}$  in which  $C_b$  is the bed friction coefficient, which may be either constant or estimated from  $C_b = g / C_z^2$ , where  $C_z = h^{1/6} / n_b$  is the Chézy constant ( $n_b$  is the Manning coefficient) and  $\tau_{wi}$  the wind stress components defined by  $\tau_{wi} = \rho C_w w_i \sqrt{w_j w_j}$  with  $C_w$  the coefficient of wind friction defined by  $C_w = \rho_a (0.75 + 0.067 \sqrt{w_j w_j}) \times 10^{-3}$  where  $\rho_a$  is the air density and  $w_i$  is the velocity of wind in the  $i$ -th direction.



**Figure 1:** Location and close-up view of Tangier Bay (right) and its triangulation mesh, where shows the locations of wastewater rivers (left).

### 3.2. Numerical method

The numerical model used in this study is the Finite Volume Shallow Water Model; it is a 2D unstructured-grid coastal ocean model that simulates water surface elevation, velocity, and transport diffusion of a tracer. The unstructured triangular cells and finite volume approach employed in the model provides geometric flexibility and computational efficiency that is well suited to simulating fine-scale features within a large domain. The model used incorporates endless limiters to provide sharp resolution numerical fluxes and slope of steep bathymetric gradients that may form in the approximate solution [1].

## 4. Numerical setup

The numerical computation has been carried out on a spatial domain that represents the Tangier's bay through a finite volume grid which consists of 7611 triangular elements and 4140 nodes (Figure 1). The bathymetry of the real domain, obtained combining several datasets, has been interpolated onto the grid. The tidal flow was forced at Atlantic Ocean side boundaries through the specification of the surface tidal elevation. We have limited our numerical modeling study to the semidiurnal components, forcing the model with the  $M_2$ ,  $S_2$  and  $N_2$  components [7]:

$$\bar{h}(t) = h_0 + \sum_{n=1}^3 A_n \cos(\omega_n t + \varphi_n) \quad (4)$$

Where,  $\bar{h}(t)$  is the water depth at time  $t$ ,  $h_0$  the initial water depth,  $A_n$  the amplitude,  $\omega_n$  the pulsation and  $\varphi_n$  the semi-diurnal component. The table 1 summarise the value of the tidal component used in this work, notice generally that the component  $M_2$  is the most dominant.

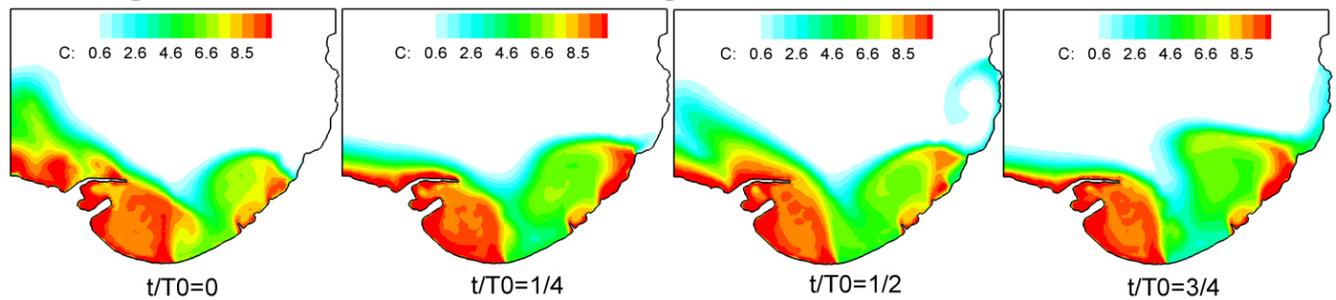
**Table 1:** Tidal component values used in numerical method.

Type	$M_2$			$S_2$			$N_2$		
Components	$A$	$\omega$	$\varphi$	$A$	$\omega$	$\varphi$	$A$	$\omega$	$\varphi$
Values	0.68	$1.4 \cdot 10^{-4}$	-67	$0.25 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	-90	$0.13 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	-56

### 5. Result and discussion

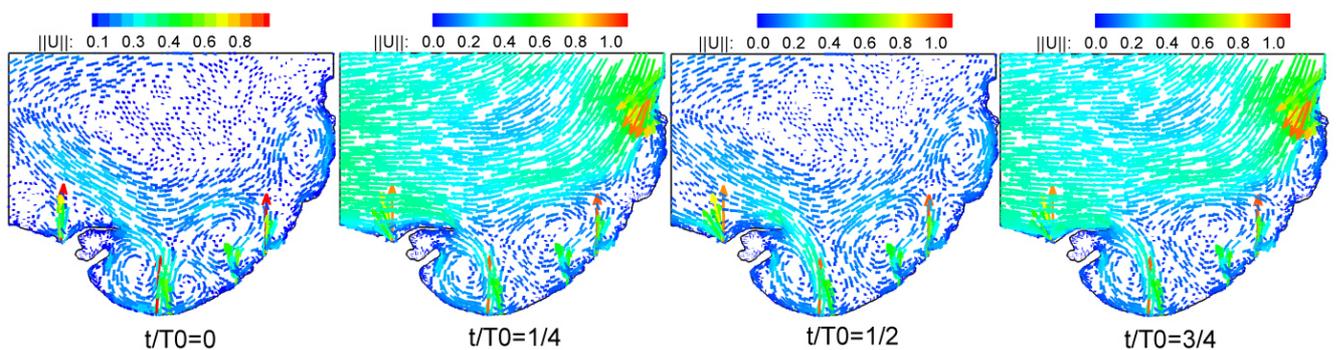
The model was run for three days with tidal forcing, in order to achieve a steady flow exchange. This time is shared between two tidal cycles, high tide and low tide of six hours each, to meet the criterion of the semi-diurnal tide that characterizes the bay, and to obtain the steady flow. Then, in third day, wastewater injection was started, it is assimilated to a passive pollutant emanating from five connected rivers.

Figure 2 shows the evolution of the wastewater dispersion through the whole bay, comprises four scenarios during a typical tidal period, the times  $t/T_0 = 0$  and  $1/4$  corresponding to low tide part and  $t/T_0 = 1/2$  and  $3/4$  to high tide. The discharges begin to enter the bay through the mouths of the rivers. The interaction between these discharges and water currents forced by the tide defines its dispersion. A global inspection of pollutant dispersion during a typical tidal period shows that during the high tide, wastewater area is confined at nearby harbor and beach and their discharges interact slowly with fresh water. Furthermore, during low tide the interaction process continues and the wastewater area expands.



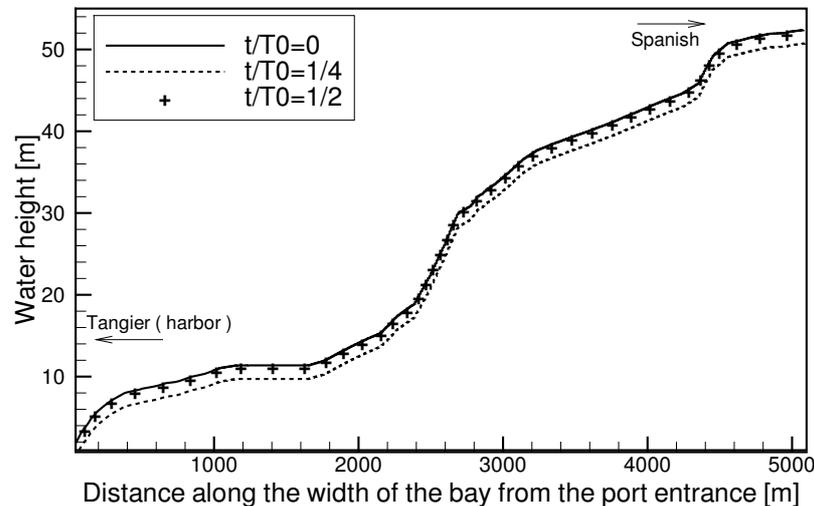
**Figure 2:** Wastewater evolution according to time, measured during a typical tidal period.

Certainly, according to the simulated velocity field showed in Figure 3, those results reveal that the change of velocity field direction during the time according to one period of the considered tides can have a direct effect on direction of propagation of wastewater. The influence of tidal currents appears from either side of the bay in terms of iso-concentrations that are essentially moved, this reflects the fact that the wastewater mass is mainly moved by the tidal flow.



**Figure 3:** Evolution of velocity magnitudes and velocity fields.

The water depth response from high-low tide is illustrated in Figure 4. Physically, the water depth is similar for the times  $t/T_0 = 0$  and  $t/T_0 = 1/2$ , and different of the time  $t/T_0 = 1/4$ . Generally, it can be concluded, that the tide also modifies water circulation within the bay, especially in the very shallow areas close to the margins. The depth-averaged velocity will significantly increase, with an even stronger coastal flow along the margins.



**Figure 4:** Numerical estimation of time evolution of water depth.

## Conclusion

In this study, an integrated numerical model is used for numerical analysis of coastal pollution process of Tangier's bay by wastewater. The model is based on coupled shallow water and advection-diffusion equations for describing, respectively, the water flow and wastewater advection-dispersion. The pollutant concentration is captured and the flow field is resolved reasonably well. The features obtained illustrate that the processes of wastewater discharges thereafter dispersed in ocean waters are influenced by the water currents, sometimes confined near bay margins and other times its areas are expands. The results of this work can prove that the proposed model is able for understanding water bay cleansing behavior.

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